SYLLABUS FOR M. SC.
IN
MATHEMATICS

Under Choice Based Credit System (CBCS)

Effective from Academic Session 2019-2020

Department of Mathematics
West Bengal State University
Berunanpukuria, P.O. - Malikapur
Barasat, Kolkata -700 126
West Bengal, India

Department of Mathematics

Semester wise Course Structures

Semester	Type of course	Paper Name	Credit	Marks	Total
I	MATCOR 01	Algebra	4	50	**Marks : 275** **Credits : 22**
	MATCOR 02	Linear Algebra	4	50	
	MATCOR 03	Real Analysis	4	50	
	MATCOR 04	Complex Analysis	4	50	
	MATCOR 05	Mechanics	4	50	
	MATAECC 01 (Practical)	Computational Techniques and Introduction to LATEX	2	25	
II	MATCOR 06	Topology	4	50	**Marks : 275** **Credits : 22**
	MATCOR 07	Functional Analysis	4	50	
	MATCOR 08	ODE and Special Functions	4	50	
	MATCOR 09	Gr. A- Numerical Analysis Gr. B- Integral Transforms	2+2=4	25+25=50	
	MATCOR 10	Differential Manifold	4	50	
	MATSEC 01 (Practical)	Computer Aided Numerical Analysis using C / Matlab/Mathematica	2	25	
III	MATCOR11	PDE and Calculus of Variations	4	50	**Marks : 300** **Credits : 24**
	MATCOR12	Nonlinear Differential Equations and Dynamical Systems	4	50	
	MATCOR13	Gr. A-Electromagnetic Theory Gr. B- Integral Equations	2+2=4	25+25=50	
	MATCOR14	Measure and Integration	4	50	
	MATDSE 01	Optional Paper*	4	50	
	MATGEC 01	Mathematics and Some Applications - I	4	50	
IV	MATCOR 15	Graph Theory / Operations Research / Fuzzy sets & Their applications	4	50	**Marks : 300** **Credits : 24**
	MATDSE 02	Advanced Paper 1**	4	50	
	MATDSE 03	Advanced Paper 2**	4	50	
	MATDSE 04	Advanced Paper 3**	4	50	
	Project/Dissertation	Project	8	100	

* List of Optional Papers :

One topic has to be chosen by a candidate from Pure Stream or Applied Stream.

Pure Stream :

1. Operator Theory and Banach Algebra
2. Number Theory and Equations over Finite Fields

Applied Stream :

1. Continuum Mechanics
2. Magneto-hydrodynamics

**List of Advanced Papers

Each year, Department will offer some courses from the following list of modules, subject to the availability of resources. Three papers (either from pure stream or applied stream) have to be chosen by a candidate from the offered modules, keeping in view the prerequisites and suitability of the combination.

Pure Stream :

1. Advanced Topology I
2. Advanced Topology II
3. Advanced Functional Analysis
4. Algebraic Topology
5. Advanced Real Analysis
6. Advanced Complex Analysis
7. Harmonic Analysis
8. Commutative Algebra

Applied Stream :

1. Quantum Mechanics
2. Plasma Dynamics
3. Theory of Waves in Solids
4. Advanced Dynamical Systems and Chaotic Dynamics
5. Solid Mechanics
6. Mathematical Biology
7. Advanced Operations Research
8. Advanced Fluid Dynamics

MATH Choice Based Subjects :

MATGEC 01 : Mathematics and Some Applications-I : Gr. A- Algebra, Metric Spaces;

Gr. B: Partial Differential Equations, Laplace Transform

PROGRAM SPECIFIC OUTCOMES

Successful completion of the two-year M.Sc. course in Mathematics will enable the students to

1. Approach and analyse the problems arising in their chosen careers in a logical manner and apply these skills to any real-life situation.
2. Apply computational and modelling skills to specific tasks, especially in the emerging and developing processes and industries.
3. Independently pursue research work in any area of Pure or Applied Mathematics; work in a group confidently and contribute significantly to any research project.
4. Acquire a systematic knowledge of fundamental aspects of various branches of Mathematics which would help them in qualifying National and State-level examinations
5. Think and analyse independently, and apply their skills in mathematical logic to any profession of their choice.
6. Take up pedagogy in Mathematics or related subjects if they are so inclined.

Semester : I

Course : MATCOR 01

Algebra : 50 Marks (4 CP)

Syllabus :

Cayley's theorem. Conjugacy classes and class equation, p-groups. Converse of Lagrange's theorem for finite abelian groups. Sylow's theorems and its applications. Direct product, finitely generated abelian groups. Solvable groups – solvability of S_n, Jőrdan-Holder Theorem.

Ideals, Principal Ideal Domain (PID). Quotient ring, isomorphism and correspondence theorems. Prime, primary and maximal ideals – examples, characterizations and their interrelations. Prime and irreducible elements. Unique Factorization Domain (UFD).

Ring with chain conditions – Noetherian rings and Artinian rings. Polynomial ring,
Semi Simple Ring, Jacobson's radical, Hilbert basis theorem.

Field extension – algebraic and transcendental extension. Splitting field, algebraic closure and algebraically closed field . Separable and normal extension. Galois field.

Galois theory (If time permits) – introduction, basic ideas and results focusing the fundamental theorem of Galois theory. Solvability by radicals.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Sylow's theorems and its applications,
ii) Jordan Holder Theorem, Solvable groups,
iii) Prime, primary and maximal ideals,
iv) Jacobsons radical, semisimple ring, Hilbert Basis Theorem,
Unique Factorization Domain,
v) Basics of Field extension & Galois theory.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Algebra, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. D.S. Malik, John M. Mordeson and M.K. Sen, Fundamentals of Abstract Algebra, McGraw Hill, International Edition, 1997.
2. Dummit and Foote, Abstract Algebra, John Wiley and Sons, Inc.
3. T. H. Hungerford, Algebra , Springer Verlag
4. John B. Fraleigh , A first course in Abstract Algebra , Narosa.
5. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd, New Delhi, 1975
6. S. Lang, Abstract Algebra, 2nd edition, Addition -Wesley .
7. Joseph Gallian, Contemporary Abstract Algebra

Course : MATCOR 02

Linear Algebra : 50 Marks (4 CP)

Syllabus :

Modules, Basic Concepts, Submodules, Quotient Modules, Isomorphisim Theorems, Correspondence Theorem, Exact Sequence, four lemma and five lemma, Simple Modules, Free modules, Modules with chain conditions(Noetherian and Artinian), Dual Modules, Fundamental Structure Theorem for Fintely Generated Modules over PID- Statement only.

Matrices and Linear Transformations, Representation of Linear Transformations between finite dimensional vector spaces by Matrices and vice versa, Linear Functionals, Dual Spaces, Dual Basis, Dimension of Quotient space.

Minimal Polynomial, Characteristic Polynomials, Diagonalization of Matrices, Reduction to Triangular Forms, Jordan Blocks, Jordan Canonical Forms, Determinant divisors and Invariant Factors, Rational Canonical Forms, Smith Normal Form Over an Euclidean Domain.

Bilinear Forms, Quadratic Forms, Hermitian Forms, Positve Definite Hermitian Forms &its Direct sum decomposition theorem, Pricipal Minor Criteron, Signature, Sylvester Law Of Inertia, Simultaneous Reduction of Pair of Forms.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge on the following :

i) Modules with chain conditions(Noetherian and Artinian), Dual Modules, Free Modules,
ii) Dual Spaces, Dual Basis, Dimension of Quotient space,
iii) Minimal Polynomial, Diagonalization of Matrices, Reduction to Triangular Forms,
iv) Jordan Canonical Forms, Rational Canonical Forms, Smith Normal Form,
v) Bilinear Forms , Quadratic Forms, Hermitian Forms,
vi) Direct sum decomposition theorem, Pricipal Minor Criteron,
vii) Sylvester Law Of Inertia, Simultaneous Reduction of Pair of Forms.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Linear Algebra, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References:

1. M. Artin, Algebra, Prentice Hall of India, 1994.
2. K. Hoffman and R. Kunze, Linear Algebra, Pearson Education (India), 2003. Prentice-Hall of India, 1991.
3. S. Lang, Linear Algebra, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1989.
4. A.R. Rao, P. Bhimashankaram, Linear Algebra. (Tata Mc-Graw Hill)
5. P. Lax, Linear Algebra, John Wiley & Sons, New York, Indian Ed. 1997
6. H.E. Rose, Linear Algebra, Birkhauser, 2002.
7. S. Lang, Algebra, 3rd Ed., Springer (India), 2004.
8. G. Strang : Linear Algebra & its Applications, Harcourt Brace Jovanichn 3rd Edition 1998.
9. B. Noble and J.W. Daniel. Applied Linear Algebra, third edition, 1988.Prentice Hall, NJ.
10. N.J. Pullman. Matrix Theory and its Applications, 1976. Marcel Dekker Inc. New York.
11. I. N. Herstein, Topics in Algebra.
12. R. Stall, Linear Algebra and Matrix Theory.
13. Evar D. Nering, Linear Algebra and Matrix Theory.
14. B. C. Chatterjee, Linear Algebra.
15. T.S. Blyth, Module Theory an approach to linear algebra- 2nd edition, Oxford Science Publication, Oxford University Press.

Course : MATCOR 03

Real Analysis: 50 Marks (4 CP)

Syllabus :

Functions of bounded variation**:** Definition and basic properties, Lipschitz condition, Jordan decomposition, Nature of points of discontinuity, Nature of points of non-differentiability, positive and negative variation and their properties.

The Lebesgue measure**:** Lebesgue Outer measure, countability, subadditivity, measurable sets and their properties, non-measurable sets, definition of Lebesgue measurable.

Measurable functions**:** Definition on a measurable set in R and basic properties, Simple Functions.

Absolutely continuous functions**:** Definition and basic properties, Deduction of the class of all absolutely continuous functions as a proper subclass of all functions of bounded variation; Characterization of an absolutely continuous function in terms of its derivative vanishing almost everywhere, property (N), every absolutely continuous function possesses the property (N).

Differentiation on R^n**:** Functions from R^n to R^m, projection functions, component functions, scalar and vector fields, open balls and open sets, limit and continuity. Derivative of a scalar field with respect to a vector, directional derivatives and partial derivatives, partial derivatives of higher order, Chain rule, Frechet derivative, matrix representation of derivative of functions, continuously differentiable functions, Implicit function theorem, inverse function theorem.

Integration on R^n**:** Integral of $f: A \to R$ when $A \subset R^n$ is a closed rectangle. Conditions of integrability. Integrals of $f: C \to R$, $C \subset R^n$ is not a rectangle, concept of Jordan measurability of a set in R^n. Fubini's theorem for integral of $f: A \times B \to R$, $A \subset R^n$, $B \subset R^n$, are closed rectangles. Fubini's theorem for $f: C \to R^n$, $C \subset A \times B$. Formula for change of variables in an integral in R^n.

Course Outcomes: Upon completion of this course, the student will be able to understand the basics of Real Analysis and improve the logical thinking.

References :

1. T. M. Apostol, *Mathematical Analysis,* Narosa Publi. House, 1985.
2. H. L. Royden, *Real Analysis*- 3rd Edn, Pearson, 1988
3. J. C. Burkil & H. Burkil, *A second Course of Mathematical Analysis,* CUP, 1980.
4. R. R. Goldberg, *Real Analysis,* Springer-Verlag, 1964.
5. I.P.Natanson, *Theory of Functions of a Real Variable,* Vol. I, Fedrick Unger Publi. Co., 1961.
6. W. Rudin, *Principle of Mathematical Analysis,* Mc Graw Hill, N.Y., 1964.
7. Charles Swartz*, Measure, Integration and Function Spaces*, World Scientific, 1994.
8. M. Spivak, *Calculus on Manifolds,* The Benjamin/Cummings Pub. comp., 1965.
9. J. R. Munkres*, Analysis on manifolds,* Addison-Wesley Pub. Comp., 1991.
10. R. Courant and F. John, *Introduction to Calculus and Analysis, Vol – II*, Springer Verlag, New York, 2004.

Course : MATCOR 04

Complex Analysis : 50 Marks (4 CP)

Syllabus :

Algebraic, Geometric and analytic preliminaries of complex numbers. Stereographic Projection, Riemann's sphere, point at infinity and its deleted neighbourhood, the extended complex plane.

Functions of a complex variable, Its Limit, Continuity and Differentiability. Analytic functions, Cauchy-Riemann equations. Branch of Logarithm, Complex integration, Winding Number or Index of a closed curve, The Cauchy-Goursat Theorem, its homotopic version (if time permits) and consequences. Cauchy's integral formula. Cauchy's integral formula for derivative, Morera's theorem, Cauchy's inequality, Liouville's theorem. Fundamental theorem of classical algebra, Schwarz Reflection Principle,.

Gauss's Mean Value Property, Maximum Modulus Theorem.

Power series, The Cauchy-Hadamard Theorem, Analyticity of Power Series, Weierstrass theorem on Uniformly convergent series of analytic functions, Uniqueness Theorem. Taylor's theorem and Laurent's theorem, Zeros of an analytic function.

Classification of Singularities, Riemann's Removal singularity thorem, Weierstrass-Casorati theorem, Limit points of zeros and poles, Classification of Singularities at infinity.

Calculus of residues, The Cauch's Residue Theorem. Argument principle, Rouche's theorem and Hurwitz's Theorem, Evaluation of definite integrals using residue theorem.

Conformal mapping, Bilinear transformation, Idea of analytic continuation.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Stereographic Projection, Riemann's sphere, point at infinity, extended complex plane,
ii) Cauchy-Goursat Theorem, Cauchy's integral formulas, Morera's theorem, Liouville's theorem,
iii) Fundamental theorem of classical algebra, Schwarz Reflection Principle, Maximum Modulus Principle,
iv) Cauchy-Hadamard Theorem, Taylor's theorem and Laurent's theorem,
v) Riemann's Removal singularity thorem, Weierstrass-Casorati,
vi) The Cauch's Residue Theorem, Argument principle and their applications,
vii) Conformal mapping, Bilinear transformation, Idea of analytic continuation.

Also there is a scope, for applying the acquired knowledge of the above concepts/ methods/ tools of Complex Analysis, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. L. V. Ahlfors **:** Complex Analysis
2. R. V. Churchill and J. W. Brown **:** Complex Variables and Applications
3. J. B. Conway **:** Functions of One Complex Variable
4. S. Ponnusamy & H Silverman, Complex Variables with Applications
5. I. Markushevich **:** Theory of Functions of a Complex Variable(Vol. I, II and III).
6. E. C. Titchmarsh **:** The Theory of Functions
7. E. T. Copson **:** An Introduction to the Theory of Functions of a Complex Variable.
8. S. Ganguly and D. Mandal, Lecture Course on Complex Analysis.

Course : MATCOR 05

Mechanics : 50 Marks (4 CP)

Syllabus :

Introduction, Kinematics for a single particle.Laws of Motion for a particle.Conservation principles. Principle of energy. Central forces and central orbits.

System of particles. Generalised Co-ordinates. Constraints, unilateral, bilateral, holonomic, schleronomic, rheonomic. Principle of Virtual Work. D'Alembert's Principle. Lagrange's equations for Holonomic and Nonholonomic Systems. Lagrange's Equation of Motion. Energy Equation for Conservative Fields. Cyclic or Ignorable Co-ordinates. Routh's Equations.

Hamilton's Equations of Motion. Calculus of Variations. Hamilton's Principle. Hamilton's and Lagrange's Equations of Motion form Hamilton's Principle. Principle of Least Action. Constants of Motion. Noether's Theorem. Conservation Laws. Dynamical systems. Liouville's theorem for conservative flow.

Motion of a Rigid Body. Euler's Theorem. Motion about a Fixed Point in it. Euler's Dynamical Equations. Motion of a symmetric top in absence of torque. Eulerian angles. Motion of a Symmetrical top under gravity. Stability of Steady Precession.

Canonical Transformations. Generating Functions. Poisson's Bracket. Jacobi's Identity. Poisson's Theorem. Jacobi-Poisson Theorem.

Hamilton-Jacobi Partial Differential Equation. Jacobi's Theorem. Hamilton's Principal Function. Hamilton's Characteristic Function. Action Angle Variables. Adiabatic Invariance.

Theory of Small Oscillations (Conservative System). Normal Co-ordinates. Oscillations under Constraints. Stationary Character of Normal Modes.

Special Theory of Relativity. Galilean Transformation and the Speed of light. Lorentz Transformation. Time dilation and length contraction. Consequences. Velocity and acceleration transformation.4 – vectors. 4-velocity.4-acceleration.4-momentum.Relativistic mass. Momentum and energy conservation in STR.Collision.4-force.

Course Outcomes :

1. Students will be able to apply the equations of motion to solve analytically the problems of motion of a single particle/a system of particle or rigid body under conservative force fields.
2. Use the Hamilton's principle for deriving the equations of motion of a system.
3. Gain knowledge of Hamiltonian system and phase planes from the point of view of mechanics.
4. Use the theory of normal modes for solving problems related to oscillations and vibrations.
5. Students will learn the basics of classical mechanics and STR required for further studies in solid and quantum mechanics.

References :

1. H. Goldstein, Classical Mechanics. Narosa Publishing House, New Delhi, (1980).

2. F. Gantmacher, Lectures in Analytical Mechanics, MIR Publishers, Moscow, (1975).

3. J. L. Synge and B.A. Griffith, Principles of Mechanics, McGraw-Hill, N.Y. (1970).

4. N. C.Rana and P. S. Joag, Classical Mechanics, Tata McGraw Hill Pub. Company Ltd., New Delhi, (1998).

5. N. H. Louis and Janet D. Finch, Analytical Mechanics, C.U.P., (1998).

6. E .T. Whittaker, A Treatise of Analytical Dynamics of Particle and Rigid Bodies, C.U.P., (1977)

7. A. S. Ramsey, Dynamics Part-II, C.U.P.

8. The Special Theory of Relativity, Morris. Medtec, (2013).

8. V. I. Arnold, Mathematical Methods of Classical Mechanics, 2nd ed., Springer-Verlag, (1997).

9. N. G. Chetaev, Theoretical Mechanics, Springer-Verlag, (1990).

10. F Chorlton, Text Book of Dynamics, CBS Publishers, (1985).

11. L. D. Landau and E.M. Lifshitz, Mechanics, 3rd ed., Pergamon Press, (1982).

12. J R Taylor- Classical Mechanics.

Course : MATAECC 01

Computational Techniques and Introduction to LaTeX (Practical) : 25 Marks (2 CP)

Syllabus :

Computational Techniques using C/Python

Programming Basics: Character set. Constants and variables data types, key words, expression, assignment statements, declaration. Arithmetic, relational and logical operators. Conditional operators.

Decision making : if statement, if-else statement, Nesting if statement, switch statement, break and continue statement, the Goto statement.

Control Statements : While statement, do-while statement, for statement.

Arrays : One-dimension, two-dimension and multidimensional arrays, declaration of arrays, initialization of one and multi-dimensional arrays.

Functions : Function declaration, Library and User defined function, Function argument. Recursion.

Programming problems for all sections above (Separate and Combined).

LaTeX

Document structure, Formatting text, math formulas and expressions, equations, tables, graphics, index, cite books, bibliography, Beamer presentation.

Course Outcomes: At the end of this course a student should be able to :

- understand the purpose of basic computer programming language,
- understand and apply control statements, implementation of arrays, functions, etc.,
- enhance ability to program writing skills for solving several real life and Mathematical problems,
- use LaTeX and develop typeset documents containing tables, figures, formulas, common book elements like bibliographies, indexes etc. and modern PDF features.

References :

1. Programming with C – Byron S. Gottfried.
2. The C Programming Language – Brian W. Kernighan, Dennis M. Ritchie.
3. Programming in ANSI C – E. Balagurusamy.
4. Let Us C : Y. Kanetkar.
5. Mastering Algorithm in C : K. Loudon.
6. A Python Book: Beginning Python, Advanced Python, and Python Exercises - Dave Kuhlman.
7. Python Essential Reference - David Beazley.
8. Latex Beginner's Guide-S. Kottwitz.

Semester : II

Course : MATCOR 06

Topology : 50 Marks (4 CP)

Syllabus :

Brief Description: Countable and uncountable sets. Axiom of choice and its equivalence. Cardinal numbers. Schroeder-Bernstein theorem. Continuum hypothesis. Zorn's lemma and well-ordering theorem. Ordinal Numbers. The first uncountable ordinal.

Topological spaces, Open and Closed sets, Bases and sub-bases. Closure and Interior – their properties and relations; Exterior, Boundary, Accumulation points, Derived sets, Adherent point, Dense set, G_δ and F_σ sets. Neighboourhoods and neighbourhood system.

Alternative methods of defining a topology in terms of Kuratowski closure operator, interior operator, neighbourhood systems.

Subspace and Induced or Relative topology. Relation of closure, interior, accumulation points etc. between the whole space and the subspace.

Continuous, open and closed maps, pasting lemma, homeomorphism and topological properties.

1^{st} and 2^{nd} countability axioms, Separability, Lindeloffness and their relationships. Characterizations of accumulation points, closed sets, open sets in a 1^{st} countable space w.r.t. sequences . Heinei's continuity criterion .

T_i spaces(i = 0, 1, 2, 3, 3½, 4, 5), their characterizations and basic properties.
Urysohn's lemma and Tieze' extension theorem (statement only) and their applications.

Connected and disconnected spaces. Connectedness on the real line. Components and quasi-components.
Compactness, its basic properties and characterizations, Alexander subbase theorem, Continuous functions and compact sets, Compactness and separation axioms . Equivalence of compactness, countable compactness and sequential compactness in metric spaces.

Product and box topology, Projection maps. Tychonoff product theorem. Separation and product spaces. Connectedness and product spaces. Countability and product spaces.

If time permits:

Identification topology and Qutient spaces.
Local Connectedness, Path- connectedness, Total disconnectedness, Zero -dimensional spaces, Extremally disconnected spaces.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge on the following :

i) Axiom of choice, Continuum hypothesis, Cardinal and Ordinal numbers,
ii) Basics of Topological spaces, Relative topology, homeomorphism and topological properties ,
iii) Alternative methods of defining a topology in terms of Kuratowski closure operator, interior operator and neighbourhood systems,
iv) Countability axioms, Heinei's continuity criterion,
v) Lower & higher separation axioms, Urysohn's lemma and Tieze's extension theorem (statement only) and their applications,
vi) Connected and disconnected spaces, path connected spaces, Compactness, Alexander subbase theorem, equivalence of various compactness in metric spaces,
vii) Product and box topology, Tychonoff product theorem,
viii) Quotient spaces, Local Connectedness, Path- connectedness, Total disconnectedness,

Also there is a scope, for applying the acquired knowledge of the above topological methods/ tools, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. General Topology by J. L. Kelley, Van Nostrand
2. General Topology by S. Willard, Addison-Wesley.
3. Topology by J. Dugundji, Allyn and Bacon.
4. Topology, A first course by J. Munkres, Prentice Hall, India.
5. Introduction to topology and modern analysis by G. F. Simmons, McGraw Hill.
6. Introduction to General topology by K. D. Joshi, Wiley Eastern Ltd.
7. General Topology by Engelking, Polish Scientific Publishers, Warszawa.
8. Counter examples in Topology by L. Steen and J. Seebach.
9. A text book of Topology by B. C. Chatterjee, S. Ganguly and M. Adhikari, Asian Books Pvt.

Course: MATCOR 07

Functional Analysis: 50 Marks (4 CP)

Syllabus :

Metric spaces, Brief discussions of continuity, completeness, compactness, connectedness. Hölder and Minkowski inequalities (statement only).

Baire's category theorem, Banach's fixed point theorem and its applications to solutions of certain systems of linear algebraic equations, Picard's existence theorem on differential equation, Implicit function theorem and Fredholm's integral equation of the second kind, Kannan's fixed point theorem.

Real and Complex linear spaces. Normed induced metric. Banach spaces, the spaces R^n, C^n, C [a, b], C_0, C, lp(n)(1≤p≤∞), lp(1≤p≤∞) and $L_2[a,b]$. Riesz's lemma. Finite dimensional normed linear spaces and subspaces, completeness, compactness criterion, Quotient space, equivalent norms and its properties.

Bounded linear operators, various expressions for its norm. Spaces of bounded linear operators and its completeness. Inverse of an operator. Linear and sublinear functionals, Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces and some of its simple applications.

Conjugate or Dual spaces, Examples, Separability of the Dual space. Reflexive spaces, weak and weak* convergence. Uniform boundedness principle and its applications. The Open mapping Theorem and the Closed graph Theorem.

Inner product spaces and Hilbert spaces, examples of Hilbert spaces, continuity of inner product, C-S inequality, basic results on Inner product spaces and Hilbert spaces, parallelogram law, Pythagorean law, Polarization identity, orthogonality, orthonormality, orthogonal complement. The Riesz representation theorem, Bessel's inequality. Convergence of series corresponding to orthogonal sequence, Fourier coefficient, Perseval identity. Riesz- Fischer Theorem.

Course Outcomes : On successful completion of this course, students will be able to appreciate how functional analysis uses and unifies ideas from vector spaces, the theory of metrics, and complex analysis. Moreover, students will be able to understand and apply fundamental theorems from the theory of normed and Banach spaces, Hilbert spaces.

References :

1. W. Rudin, *Functional Analysis*, Tata McGraw Hill.
2. B.V. Limaye, *Functional Analysis*, Second Edition, New Age – International limited, Madras.
3. G. Bachman & L. Narici- *Functional Analysis*, Academic Press, 1966.
4. N. Dunford & J. T. Scwarz – Linear operators, Vol – I & II, Interscience, New York, 1958.
5. L. V. Kantorvich and G.P. Akilov-*Functional Analysis*, Pergamon Press, 1982.
6. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern, 1989.
7. I. J. Madox- *Elements of Functional Analysis*, Universal Book Stall, 1992.
8. A. H. Siddiqui, K. Ahmed and P. Manchanda, *Introduction to Functional Analysis with applications*, Anshan Publishers, 2007.
9. A. E. Taylor- *Functional Analysis*, John wiley and Sons, New York, 1958.

Course: MATCOR 08

Ordinary Differential Equations and Special Functions: 50 Marks (4 CP)

Syllabus :

First- order equations: Well-posed problems, existence and uniqueness of solution of the first order initial value problem .Cauchy Peano existence theorem. Lipschitz condition. Picard's method of successive approximations. Picard- Lindeloeff theorem. Continuation of solution. Dependence on parameters and on initial value.

Existence and uniqueness theorems for systems of first order differential equations and higher order ordinary differential equations.

Theory of Linear systems and n th order linear ODE.

System of linear homogeneous and non-homogeneous differential equations. Fundamental matrix. Exponential matrix function and their properties. Method of solving systems of linear ordinary differential equations by fundamental matrix and exponential matrix function. Equations with constant coefficients and periodic coefficients.

Nth order linear ordinary differential equations. Method of variation of parameters.

Linear Autonomous System, Phase Plane Analysis, Equilibrium Points, Classification of equilibrium points, Stablility of equilibrium points .

Adjoint and self-adjoint linear differential equations: Abel's identity, oscillatory solutions. Sturm's separation and comparison theorems.

Eigenvalue problems , Sturm – Liouville problem, solution by Green's function. Eigenvalues and Eigenfunctions.Properties.Fourier Seriesexpansion in terms of eigenfunctions.

Special Functions: Concepts of ordinary and singular points of a second order linear differential equation in a complex plane, Fuch's theorem, Solution at an ordinary point, Regular singular point, Frobenius Method, Solution at a regular singular point, Series solutions of Legendre and Bessel equations.

Legendre polynomial: Generating function, Schlaffli's integral,Rodrigue's formula, recurrence relations, orthogonality property, expansion of a function in a series of Legendre polynomials. Bessel function and its properties.

Course Outcomes:

1. Students will learn about existence and uniqueness of solutions and Picard's method of approximation . This can be directly applied for a numerical approximation.
2. Knowledge of the properties of eigenvalues and eigenfunctions will be useful in studying Mathematical physics.
3. An acquaintance with special functions will be useful for students interested in research in continuum mechanics or theoretical physics.
4. An acquaintance with special functions will be useful for students interested in research in continuum mechanics or theoretical physics.
5. Introductory ideas of phase plane analysis and stability can be utilised by students while studying dynamical systems or mathematical biology.
6. Students will be able to solve/analyse odes arising in different areas of physics.

References :

1. E. A. Coddington, An introduction to ordinary Differential Equations, Prentice- Hall of India.

2. E. A. Coddington and N Levinson Theory of Ordinary Differential Equations .McGraw Hill 1955.

3. E. L. Ince Ordinary Differential Equations- (Dover) 1956.

4. S L Ross, Ordinary Differential Equations, Wiley India, 2004.

5. G. Birkhoff & G. Rota, Ordinary Differential Equation , Wiley, 1989.

6. J. C. Burkill, The Theory of Ordinary Differential Equations, Oliver & Boyd, London, 1968.

7. E. A. Coddington and R Carlson.Linear Ordinary Differential Equations, PHI Delhi, 2013.

8. Lawrence Perko, Differential Equations and Dynamical Systems, Springer, 2001.

9. R. P. Agarwal & R. C. Gupta, Essentials of Ordinary Differential Equations, MGH, 1993.

10. N. N. Lebedev, Special Functions and Their Applications, PHI, 1972.

11. E. D. Rainville, Special Functions, Macmillan, 1971.

12. G. F. Simmons, Differential Equations, TMH, 2006.

13. I. N. Sneddon, Special Functions of mathematical Physics & Chemistry, Oliver & Boyd, London, 1980.

Course: MATCOR 09

Gr. A - Numerical Analysis ; Gr. B - Integral Transforms: 50 Marks (4 CP)

Syllabus :

Gr-A : Numerical Analysis (25 Marks)

Numerical Solution of System of Linear Equations: Triangular factorisation methods, Iterative methods : Jacobi method, Gauss-Seidel method and Gauss Jacobi method and their convergence ,diagonal dominance, Successive-Over Relaxation (SOR) method, Ill- conditioned matrix.

Eigenvalues and Eigenvectors of Real Matrix: Power method for extreme eigenvalues and corresponding eigenvectors, Gerschgorin's circle theorem.

Solution of Non-linear Equations: Newton-Raphson and secant method , rate of convergence , General iterative method for the system : $x = g(x)$ and its convergence. Non-Linear Systems of Equations: Newton's method

Polynomial Interpolation: Weirstrass's approximation theorem (Statement only), Hemite interpolation, Cubic spline interpolation.

Numerical Integration: Newton-Cotes formulae, Romberg integration.

Numerical Solution of PDE : Finite Difference Methods, Heat equation, Crank-Nicolson method, five point formula for solving Laplace and Poission equations. Wave equation: Explicit and Implicit method of solving Cauchy problem.

Stability of methods and solutions.

References :

1. S.D. Conte and C. DeBoor, Elementary Numerical Analysis: An Algorithmic Approach, McGraw Hill, N.Y., 1980.
2. A. Ralston, A First Course in Numerical Analysis, McGraw Hill, N.Y. , 1965.
3. A. Ralston and P. Rabinowitz, A First Course in Numerical Analysis, McGraw Hill, N.Y., 1978.
4. K.E. Atkinson, An Introduction to Numerical Analysis, John Wiley and Sons, 1989.
5. S.D. Conte and C. DeBoor, Elementary Numerical Analysis: An Algorithmic Approach, McGraw Hill, N.Y., 1980.
6. W.F. Ames, Numerical Methods for PDEs, Academic Press, N.Y., 1977.
7. L. Colatz, Functional Analysis and Numerical Mathematics, Academic Press, N.Y., 1966.
8. Jain, Iyengar and Jain, Numerical methods for scientific and Engineering Computation, New Age International Pub.
9. F.B.Hilderbrand, Introduction to Numerical Analysis, Dover Publication.

Gr-B : Integral Transforms (25 Marks)

The Fourier Transform:

Fourier Integral Theorem. Derivation of Fourier transform from Fourier series, Properties of Fourier transform, Convolution, Transform of derivatives. Fourier cosine and sine transforms. Inverse Fourier transform. Parseval's Identity. Finite Fourier Transform. Application to solving ordinary and partial differential equation.

The Laplace transform:

Definition and properties. Sufficient conditions for the existence of Laplace Transform. Transform of derivatives. Convolution theorem. Inversion of Laplace Transform. Evaluation of inverse transforms by residue. Initial and final value theorems. Heaviside expansion theorem. Applications of Laplace transform.

The Z-Transform:

Definition and properties. Z-transform of some standard functions. Inverse Z-transforms. Applications.

Course Outcomes: After completion of the course, the student is expected to :

- understand basic theories of numerical analysis,
- formulate and solve numerically problems from different branches of science,
- grow insight on computational procedures,
- learn theory and properties of Fourier transform, Laplace Transform and Z-Transform and their applications to relevant problems.

References :

1. I.N. Sneddon - The Uses of Integral Transforms.
2. C.J. Tranter - Integral Transforms.
3. L.Debnath & D.Bhatta - Integral Transforms and Their Applications.
4. J. L. Schiff - The Laplace Transform (Springer)
5. R.L. Bracewell -The Fourier Transforms and Its Applications (McGraw-Hill).
6. E.J. Watson - Laplace Transforms and Application, Van Nostland Reinhold Co. Ltd., 1981.
7. E.I. Jury - Theory and Application of Z-Transform, John Wiley and Sons, N.Y.

Course: MATCOR 10

Differential Manifold : 50 Marks (4 CP)

Syllabus :

Differentiable manifolds: basic notions; the effects of second countability and Hausdorffness; tangent and cotangent spaces; submanifolds; consequences of the Inverse Function Theorem; vector fields and their flows; the Frobenius Theorem; Sard's theorem.

Differential forms: Multilinear algebra; tensors; differential forms; the de Rham complex and its behaviour under differentiable maps; the Lie derivative; differential ideals.

Lie groups: Lie groups; Lie algebras; homomorphisms; Lie subgroups; coverings of Lie groups; the exponential map; closed subgroups; the adjoint representation; homogeneous manifolds.

Integration on manifolds: orientation; the integral of differential forms on differentiable singular chains; integration of differential forms of top degree on an oriented 3 differentiable manifold; the theorems of Stokes; the volume form on an oriented Riemannian manifold; the divergence theorem; integration on a Lie group.

de Rham cohomology: definition; real differentiable singular cohomology; statement of the de Rham theorem; the Poincar lemma.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) tangent and cotangent spaces; submanifolds,
ii) vector fields and their flows; the Frobenius Theorem,
iii) multilinear algebra, differential forms, the Lie derivative,
iv) Lie groups and Lie algebras,
v) Integration on manifolds, theorems of Stokes, integration on a Lie group,
vi) de Rham cohomology.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Differentiable manifolds to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions.

References :

1. M. Spivak; A Comprehensive Introduction to Differential Geometry, Vols I-V; Publish or Perish, Inc. Boston, 1979
2. J.A. Thorpe Elementary topics in Differential Geometry, Under - graduate Texts in Mathematics, Springer – Verlag, 1979.
3. Kobayashi. S. and Nomizu. K. Foundations of Differential Geometry, Interscience Publishers, 1963
4. F. W. Warner, Foundations of differentiable manifolds and Lie groups.
5. Christian Br; Elementary Differential Geometry; Cambridge University Press, 2011.
6. I. Madsen and J. Tornehave, From calculus to cohomology, Cambridge University Press.

<u>Course : MATSEC 01</u>

Computer Aided Numerical Analysis using C/ Matlab/ Mathematica (Practical) :

25 Marks (2 CP)

Syllabus :

Programming Problems :

1. Cubic spline interpolation.
2. Gauss Elimination Method for a System of Linear Equations.
3. Newton's Method for a System of Nonlinear Equations.
4. Inverse of a matrix.
5. Integration by Romberg's method.
6. Largest Eigen values of a real matrix by power Method.
7. Numerical Solutions of Ordinary Differential Equations for Initial Value Problems : (a) Picard's Formula, (b) Adams-Bashforth method, (c) Milne's predictor–corrector method.
8. Finite Difference Method for PDE – Elliptic Type PDE, Parabolic Type PDE, Hyperbolic Type PDE.

Course Outcomes: At the end of this course a student should be able to :

- solve different type of numerical problems,
- understand better relevant theoretical concepts,
- apply programming skills in interdisciplinary areas such as biological system, physical system etc.,
- analyze data set of various size and interpret outcomes helping her/him to compete in the financial sector.
- apply programming skills in graphics animation, computerized abstract art.

References :

1. Computing methods - Berzin and Zhidnov.
2. Analysis of Numerical methods - Isacson and Keller.
3. A first course in Numerical Analysis - Ralston and Rabinowitz.
4. Numerical solution of differential equations - M. K. Jain.
5. Numerical solution of partial differential equations- G. D. Smith.
6. Theory and Problems of Numerical Analysis - F. Scheid
7. Applied numerical methods using Matlab, W. Y. Yang, W. Cao, T-S Chun and J. Morris, Wiley-Interscience, John Wiley & Sons, 2005.

Semester : III

Course : MATCOR 11

Partial Differential Equations and Calculus of Variations : 50 Marks (4 CP)

Syllabus :

Origins of Partial Differential Equations(PDE). Linear and non- linear PDE. Cauchy's method of characteristics , Charpit's method, Jacobi's method.

Second order PDE with constant and variable coefficients. Reduction to canonical forms and Classification, characteristic curves. Well-posed and ill-posed problems. Non linear PDE of second order.

Wave equation: vibrations of strings, D'Alembert's solution, Riemann's method, Solution by separation of variables, Transverse vibrations of membranes.

Laplace Equation: Equipotential surfaces, Boundary value problems, Maximum-minimum principles, The Cauchy problem, Stability of the solution. Theory of Green's function.

Diffusion equation: Boundary value problems, variables separable solution. Duhamel's Principle.

Solution of linear partial differential equations by Lie algebraic method.

Calculus of Variations

Linear functional, Euler equation, The Brachistochrone problem: Cycloid, Geodesic, Several dependant variables : Lagrange's equations, Isoperimetric problem, Variational problems : parametric form , with moving boundaries, least action principle.

Course Outcomes: At the end of this course a student should be able to :

- learn to solve different types of PDE,
- test the stability of the solution,
- apply PDE to problems of geometry and physics,
- understand basic theories of calculus of variations,
- formulate and solve problems from allied branches of science.

References :

1. Sneddon I.N. : Elements of Partial Differential Equations, Mcgraw Hill.
2. Petrovsky I.G. : Lectures on Partial differential equations.
3. Courant and Hilbert : Methods of Mathematical Physics, Vol – II.
4. L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society.
5. F. John, Partial Differential Equations, Narosa.
6. Williams W.E. : Partial Differential Equations.
7. Miller F.H. : Partial Differential Equations.
8. K.S. Rao, Introduction to partial differential equations, Prentice Hall, New Delhi, 1997.
9. M. Krasnov et. al., Problems and exercises in the calculus of variations, Mir Publishers.
10. A. S. Gupta, Calculus of Variations with Applications, Prentice Hall, India.
11. Zafar Ahsan, Differential Equation and their applications, PHI Learning , New Delhi.
12. I. M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall Inc.
13. R. Weinstock- Calculus of Variations, Dover Publications.

Course : MATCOR 12

Nonlinear Differential Equations and Dynamical Systems: 50 Marks (4 CP)

Syllabus :

System of ODE's Autonomous System, Phase Plane Analysis, Equilibrium Points, Classification of equilibrium points, Stablility of equilibrium points. Nonlinear autonomous systems. Flow diagram, Phase portrait, Isocline. Fixed points and their nature. stability, asymptotic stability, Linearization about a critical point. Liapunov function.

Conservative systems. Hamiltonian systems. Index of an equilibrium point. The index at infinity. The phase diagram at infinity. Homoclinic and heteroclinic paths. Limit cycles and other closed paths.

Averaging methods. Energy balance method for limit cycles. Amplitude and frequency estimates. Nearly-periodic solutions. Periodic solutions and Harmonic balance method.

Perturbation methods for Duffing;s equation. Peridic solution of autonomous systems. Lindstedst's method. Singular perturbation. Lighthills method.

Stability. Poincare and Lyapunov stability. Solutions and paths, linear systems, zero solutions of nearly linear systems.

The existence of periodic solutions. The Poincare Bendixson theorem.

Simple bifurcations. The saddle-node, transcritical and pitchfork bifurcation. Hopf bifurcation.

Manifolds. Stable Manifold and Centre manifold theorem.

Course Outcomes:

1. On the completion of this course students will be able to study the nature linear stability and general stability of critical points and solutions ; also investigate the existence of periodic solutions ; and identify a bifurcation through change of parameters ; further, have a basic idea of perturbation methods.

2. These methods can be applied by the students to study problems of population biology and nonlinear wave propagation.

References :

1. Nonlinear Ordinary Differential Equations - D. W. Jordan, P. Smith, OUP, 2007.
2. Nonlinear Differential Equations and Dynamical Systems - F. Verhulst, Springer.
3. Nonlinear Dynamics and Chaos - Steven H Strogatz. Levant Books.Kolkata 2007.
4. An Introduction to dynamical systems - D. K. Arrowsmith and C. M. Place. CUP 1990.
5. Differential Equations and Dynamical Systems - L.Perko. Springer, NY, 1991.
6. Nonlinear Oscillations, Dynamical systems, and bifurcation of vector fields - J. Guckenheimer and P. Holmes. Springer NY, 1983.
7. Nonlinear Ordinary Differential Equations - R Grimshaw, Blackwell, Oxford, 1990.

Course : MATCOR 13

Gr. A-Electromagnetic Theory ; Gr. B- Integral Equations: 50 Marks (4 CP)

Syllabus :

Gr. A-Electromagnetic Theory (25 Marks)

Electrostatics: Coulomb's law, Electric Field, Divergence and Curl of Electrostatic Fields, Gauss' law, Electric Potential: Poisson and Laplace equation, Work and Energy, Conductors.

Electric Fields in Matter: Polarization, Electric Displacement, Linear Dielectrics.

Magnetostatics: Lorentz Force Law, Steady currents, Biot-Savart Law, Divergence and Curl of B, C Magnetic Vector Potential. Magnetic Fields in Matter: Field of a Magnetized Object, Ampere's Law in Magnetized Material, Linear and Nonlinear Media.

Electromagnetic Induction: Faraday's Law, Maxwell's Equations. Conservation Laws, Continuity Equation, Poynting's Theorem. Newton's Third Law in Electrodynamics, Maxwell's Stress Tensor, Conservation of Momentum.

Electromagnetic Waves in Vacuum and Matter, Fresnel's equations, Absorption and Dispersion, Guided Waves. Coulomb Gauge and Lorenz Gauge, Jefimenko's Equations, Dipole radiation, Radiation reaction.

Relativistic electrodynamics : Einstein's postulates , Lorentz transformation, Magnetism as a Relativistic phenomenon, Field transform, Field tensor, Relativistic potential.

References :

1. D.J. Griffiths, Introduction to Electrodynamics, Prentice Hall, New Delhi.
2. L. D. Landan and E. M. Lifshitz, The classical Theory of Fields.
3. A. Sommerfield, Electrodynamics.
4. J.D. Jackson, Classical Electrodynamics.
5. J. H. Jeans, Mathematical Theory of Electricity and Magnetism, Cambridge University Press.
6. V. S. A. Ferraro, Electromagnetic Theory, Athlone Press, London.
7. I. E. Irodov, Basic laws of Electromagnetism, CBS.

Gr. B - Integral Equations (25 Marks)

Definition of Integral Equation and their classification. Reduction of differential equation to integral equation and vice-versa. Eigen values and Eigen functions.

Existence and uniqueness of solutions of Fredholm and Volterra integral equations of second kind. Solution by the method of successive approximations, series solution. Iterated kernels. Reciprocal kernels. Neumann series. Solution of integral equations with separable kernels. Solution of Volterra integral equation of first kind.

Fredholm theorems and Fredholm Alternative. Hilbert-Schmidt theory of integral equations for symmetric kernels.

Singular Integral equation, Solution of Abel’s Integral equation. Solution of Volterra equation of convolution type by Laplace transform.

Course Outcomes: After completing this course, the student will be able to:

- build up strong application capability of graduate level mathematics,
- understand and apply the basic theories of electromagnetism,
- get an exposure to the Einstein’s Theory of Relativity,
- grow interest in electrical engineering,
- distinguish between differential and integral equations,
- understand the theory of existence and uniqueness of solutions of linear integral equations,
- find solutions of linear integral equations of first and second type (Volterra and Fredhlom) and singular integral equations using several techniques.

References :

1. R. P. Kanwal - Linear Integral Equation – Theory and Techniques, Academic Press, New York, 2012.
2. W.V. Lovitt - Linear Integral Equations, Dover, New York.
3. A. Wazwaz - A first course in integral equations, World Scientific, 1997
4. F. G. Tricomi - Integral Equations, Dover.
5. S. G. Mikhlin - Integral Equations, Pergamon Press, 1960.

Course : MATCOR 14

Measure and Integration: 50 Marks (4 CP)

Syllabus :

Outer Lebesgue Measure m^* in the Euclidean line and its Properties. Outer measure $\boldsymbol{\mu}^*$ on S, where S is a space; the concept of μ-measurable sets with the help of $\boldsymbol{\mu}^*$. Necessary and sufficient condition for μ-measurability. Properties of **μ**-measurable sets. The structure of μ-measurable sets-the concept of σ-algebra; the σ-algebra of Lebesgue measurable sets.

Properties of Lebesgue measure, Vitali's theorem: The existence of an non-measurable set in the Euclidean line . The Borel sets & Lebesgue measurable sets- a comparison

μ-measurable functions, their properties; Characteristic functions, Simple functions. Theorem relating to the non negative μ-measurable function as a limit of a monotonically increasing sequence of non negative simple μ-measurable functions.

Lebesgue Integration : Integration for simple functions and for Extended real valued **μ**-measurable functions; The countable additivity of the set of function ν_f on **M** defined by $\nu_f(M) = \int_M f$, for each set M Є **M** , the **σ-** algebra of μ-measurable sets, for a nonnegative **μ**-measurable function f ;

Lebesgue's monotone convergence theorem and its applications, Fatou's lemma, Lebesgue's dominated convergence Theorem.

Necessary & Sufficient condition of Riemann integrability via measure; interrelation between the two modes of integration.

The Concept of L^p-spaces; Inequalities of Holder and Minkowski; Completion of L^p-spaces.

Convergence in Measure, Almost Uniform Convergence, Pointwise Convergence a.e and their Characterizations; Convergence Diagrams, Counter Examples. Egoroff theorem.

Lebesgue Integral in the Plane. Product σ-algebra. Product Measure. Fubini's Theorem.

If time permits :

Signed Measure and the Hahn Decomposition; The Jordan Decomposition. The Radon-Nikodym Theorem.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Lebesgue measure, Vitali's theorem concerning existence of non-measurable sets,
ii) measurable functions, Theorem relating to non negative μ-measurable function as a limit of a monotonically increasing sequence of non negative simple μ-measurable functions,
iii) Lebesgue's monotone convergence theorem and its applications, Fatou's lemma, Lebesgue's dominated convergence Theorem,
iv) Interrelation between Riemann & Lebesgue integration,
v) Concept of L^p-spaces and its completeness,
vi) Characterizations of Convergence in Measure, Almost Uniform Convergence, Egoroff theorem,
vii) Product Measure. Fubini's Theorem,
viii) Signed Measure and the Hahn Decomposition, Radon-Nikodym Theorem.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Measure and Integration, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. P. R. Halmos, Measure Theory, Von Nostrand, New York, 1950.

2. E. Hewitt & K. Stromberg, Real and abstract Analysis, Third edition, Springer-.

3. G. D. Barra, Measure Theory & Integration, Wiley Eastern Limited, 1987.

Verlag, Heidelber & New York, 1975.

4. W. Rudin, Real and Complex Analysis, Tata McGraw- Hill, New York, 1987.

5. I. K. Rana, An introduction to Measure & Integration, Narosa Publishing House, 1997.

6. H. L.Royden, Real Analysis, Macmillan Pub. Co. Inc, New York, 1993.

7. J. F. Randolf, Basic Real and Abstract Analysis, Academic Press, New York, 1968.

8. C. D. Aliprantis and Owen Burkinshaw, Principles of Real Analysis, Academic Press, 2000.

9. K. R. Parthsarathy, introduction to Probaility and Measure, Macmillan Company of India Ltd., Delhi, 1977.

10. R. B. Bartle, Elements of Real Analysis.

Course : MATDSE 01

(Optional Paper*)

Pure Stream

1. Operator Theory and Banach Algebra: 50 Marks (4 CP)

Syllabus :

Dual spaces, Representation Theorem for bounded Linear functionals on C[a,b] and L_p spaces, Dual of C[a,b] & L_p spaces, weak & weak*convergence, Reflexive spaces.

Bounded Linear Operators, Uniqueness Theorem, Adjoint of an Operator and its Properties; Normal, Self Adjoint, Unitary, Projection Operators, their Characterizations & Properties. Orthogonal Projections, Characterizations of Orthogonal Projections among all the Projections. Norm of Self Adjoint Operators, Sum & Product of Projections, Invariant Subspaces. Sesquilinear functionals on linear spaces and on Hilbert spaces, generalization of Cauchy-Schwarz inequality.

Spectrum of an Operator, Finite Dimensional Spectral Theorem, Spectrum of Compact Operators, Spectral Theorem for Compact Self Adjoint Operators (statement only).

Algebra and some properties of the space C(X), Stone-Weierstrass Theorem.
Banach Algebra, Banach Sub Algebra, Identity element, invertible elements, existence and representation of the inverse of e-x, resolvent set and resolvent operator, analytic property of the resolvent operators, compactness of spectrum, non-emptiness of the spectrum. Division Algebra, Gelfand-Mazur Theorem. Topological divisors of zero. Spectral radius and its properties, spectral mapping theorem for polynomials. Complex homomorphism, Gleason-Kahane-Zalazko Theorem, Commutative Banach Algebra, Ideals, maximal ideals, Quotient space as a Banach Algebra under certain conditions. Gelfand theory on representation of Banach Algebra, Gelfand transform, weak Topology, weak* Topology, Gelfand Topology, Banach Alaoglu Theorem. Quotient algebra, Banach * algebra, B* algebra, Gelfand Naimark Theorem.

Course Outcomes : Students will be able to understand the fundamentals of spectral theory, and appreciate some of its power. Students will have the knowledge and skills to apply problem solving using functional analysis techniques applied to diverse situations in physics, engineering and other mathematical contexts.

References :

1. Rudin, Functional Analysis.
2. Schaffer, Topological Vector Spaces.
3. Bachman & Narici, Functional Analysis.
4. Kryszic, Functional Analysis.
5. Diestal, Applications of Geometry of Banach Spaces.
6. Horvat, Linear Topological spaces.
7. Brown and Page, Elements of Functional Analysis.

2. Number Theory and Equations over Finite Fields: 50 Marks (4 CP)

Syllabus :

Prime Numbers and Unique Factorization, Primes in Arithmetic Progressions, Euclids Alghorithm, Wilsons Theorem, Linear congruence; ax ≡ b(mod n), Sums of Two Squares, Chinese Remainder Theorem, Euler's Theorem.

Primitive roots modulo n, the existence of primitive roots, applications of primitive roots, Structure of U(Z/nZ), The equation $x^n \equiv a \pmod m$ (n^{th} Power residues), The ring of Gaussian integers Z[i], Integral Binary Quadratic forms $aX^2 + bXY + cY^2$,

Quadratic Reciprocity Laws: Legender Symbol and a Gauss Sum, proof of the law of quadratic reciprocity.

Equations over Finite Fields : Finite Fields, Gauss and Jacobi Sums,: Chevalley-Warning Theorem, Quadratic Forms over finite fields and their reduction to the equation $a_1 x^{l_1} + a_2 x^{l_2} + \cdots + a_r x^{l_r}$

$= b$ over F_q.

Construction of p-adic numbers, ring of p-adic integers, some appliations.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Wilsons Theorem, Linear congruence; ax ≡ b(mod n),
ii) Chinese Remainder Theorem, Euler's Theorem,
iii) applications of primitive roots, Structure of U(Z/nZ),
iv) law of quadratic reciprocity,
v) Equations over Finite Fields: Chevalley-Warning Theorem,
vi) Quadratic Forms over finite fields,
vii) p-adic numbers and its applications.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Number Theory and Equations over Finite Fields, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions.

References :

1. D. M. Burton; Elementary Number Theory; Wm. C. Brown Publishers, Dulreque, Lowa, 1989
2. Stilwell, J., Elements of Number Theory, Sprinter UTM 2003.
3. Gareth A. Jones and Mary Jones J., Elementary Number Theory, Springer SUMS 2005.
4. Neal Koblitz; A course in number theory and cryptography; Springer-Verlag, 2nd edition.
5. Ireland K. and Rosen M., A Classical Introduction to Modern Number Theory, Springer GTM 2004.
6. Lidl R. and Niedrreiter H., Finite Fields, Encyclopedia of Mathematics and its Applications 20, Cambridge 1997.
7. Richard A Mollin; Advanced Number Theory with Applications; CRC Press, A Chapman & Hall Book.
8. Flath D.E., Introduction to Number Theory, John Wiley & Sons 1989.

Applied Stream

1. Continuum Mechanics : 50 Marks (4 CP)

Syllabus :

Introduction. Idea of continuum: Continuum hypothesis, Continuous media, Body, Configuration, continuum motion as a real continuous map. Material and spatial time derivatives.

Theory of deformation and strain: Deformation and flow, Lagrangian and Eulerian descriptions, Deformation gradient tensors, Finite strain tensor, Finite strain components in rectangular Cartesian coordinates, Small deformation, Infinitesimal strain tensor, Infinitesimal strain components, Geometrical interpretation of infinitesimal strain components, Strain quadric of Cauchy, Principal strains, Strain invariants, Compatibility equations for linear strains.

Rate of strain tensors-its principal values and invariants, Rate of rotation tensor, vorticity vector, velocity gradient tensor.

Theory of stress: Forces in a continuum, Stress tensor, Equations of equilibrium, Symmetry of stress tensor, Shearing and normal stresses, Stress quadric of Cauchy and its properties. Maximum shearing stress, Principal stresses and principal axes of stresses, Invariants of stress tensors, Stress compatibility equations.

Motion of a continuum: Principle of conservation of mass, The continuity equation, Principle of conservation of linear and angular momentum, conservation of energy.

Theory of elasticity: Ideal materials, Classical elasticity, Generalized Hooke's Law, Isotropic and anisotropic materials, Constitutive equation for isotropic elastic solid, and anisotropic solids. Elastic modulii, Strain-energy function, Physical interpretation.

Motion of fluid: Path lines, stream lines and streak lines, Material (Bounding) surface, Lagrange's criterion for material surface.

Constitutive equations for Newtonian Fluid. Stress and rate of strain relation.

Irrotational motion of fluid: Irrotational motion, Velocity potential, Circulation, Kelvin's circulation theorem, Kelvin's theorem of minimum kinetic energy.
Equation of motion of inviscid fluid: Inviscid incompressible fluid, Constitutive equation, Euler's equation of motion & its vector invariant form, Bernoulli's equation and applications to some special cases, Helmholtz's equation for vorticity, Impulsive generation of motion and some properties, Navier-Stokes' Equations, Boundary Conditions .

Two dimensional irrotational flow of incompressible fluid. Stream function.Velocity potential. Complex potential. Sources and sinks. Image systems. Complex potential for elementary flows.

Course Outcomes:

- The students will learn a new approach, namely, the continuum approach for both solid and fluid motion.
- Students will learn the general forms of balance laws and energy equation.
- This course will prepare the students for further courses on fluid and/or solid dynamics.

References :

1. A. C. Eringen - Mechanics of Continua, Wiley, 1967.
2. I. S. Sokolnikoff - Mathematical theory of Elasticity, Tata Mc Grow Hill Co., 1977.
3. S. K.Bose - Continuum Mechanics Theory, Narosa, 2017.
4. S.W. Yuan - Foundations of Fluid Mechanics, Prentice – Hall International, 1970.
5. J. L. Bansal - Viscous Fluid Dynamics, Oxford and IBH Publishing Co., 1977.
6. D. S. Chandrasekharaiah and L. Debnath- Continuum Mechanics, Academic Press, 1994.
7. J. Milne-Thomson -Theoretical Hydrodynamics.

2. Magneto-hydrodynamics : 50 Marks (4 CP)

Syllabus :

Basic ideas of electro-magnetic fields, basic laws. Maxwell's equation,- in vacuum , in matter, physical significance, boundary conditions ; Energy transfer and Poynting theorem.

Equation of motion of a conducting fluid, simplification of MHD equations using dimensional consideration (i.e. MHD approximations), magnetic Reynold's number, Alfven's theorem, the magnetic body force, Ferraro's law of isorotation, Non-dimensional form of the equation.

Steady laminar flow of a viscous conducting fluid between parallel walls in the presence of a transverse magnetic field (i.e. Hartmann flow), Two dimensional MHD equations, Couette flow, Transient Couette flow, Flow through a rectangular duct. Unsteady incompressible flows, Rayleigh's problem.

Magnetohydrostatics, equilibrium configurations, Pinch effect, force-free fields, non-existence of force free field of finite extent. General solution for a force free field.

The generalized Hugoniot condition. The compressive nature of magneto hydrodynamic shocks. Mach number, Subsonic and supersonic flows. Sub and super Alfvenic waves.

Course Outcomes: At the end of this course a student should be able to :

- describe the properties of Magneto-hydrodynamic equations,
- explain MHD waves,
- apply the MHD equations to a number of astrophysical problems as well as to problems related to laboratory physics.

References :

1. J. A. Shercliff – A text book of Magnetohydrodynamics, Pergamon Press, 1965.
2. T. G. Cowling- Magneto Hydrodynamics.
3. A. Jeffrey – Magnetohydrodynamics, Oliver & Boyd, 1966.
4. V. C. A. Ferraro & C. Plumpton, An introduction to Magneto-Fluid Mechanics, Clarendon Press, 1966.

Course: MATGEC 01

Mathematics and Some Applications - I : 50 Marks (4 CP)

Syllabus :

Gr. A- Algebra, Metric Spaces (25 Marks)

Algebra

Group, Subgroups, Normal Subgroups, Abelian Groups, Cyclic groups, Elementary properties and Examples of Normal Subgroups and Cyclic groups, Permutations, Symmetric Groups, Lagrenge's Theorem, Cayley's Theorem (Statement only).
Ring, Sub Ring, Field, Sub Field, Elementary properties and Examples of Rings and Fields.

Metric Spaces

Definition and examples. Open and closed balls, neighbourhood, open set, interior of a set. Limit point of a set, closed set, diameter of a set, subspaces, dense sets, separable spaces. Sequences in Metric Spaces, Cauchy sequences and its properties. Complete Metric Spaces, Statement of Cantor's theorem.

Gr. B: Partial Differential Equations, Laplace Transform (25 Marks)

Partial Differential Equations

First Order Equations: Geometrical Interpretation. Lagrange's Method, Charpit's Method, Classification of second order PDE. Reduction to canonical forms. Variables separable solutions to the fundamemtal PDEs of physics.

Laplace Transform

Definition and basic properties. Laplace Transform of some elementary functions. Laplace transform of the derivatives. Convolution theorem. Inverse Laplace transform. Application to solving differential equations.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, demonstrate and apply the acquired knowledge of the following :

i) Basics of Group, Subgroups, Normal Subgroups, Abelian Groups, Cyclic groups,
ii) Symmetric Groups, Lagrenge's Theorem, Cayley's Theorem,
iii) Ring, Sub Ring, Field, Sub Field,
iv) Basic game theory and graph theory,
v) Inner Product Space, Orthogonal sets and Bases, Eigenvalues, Eigenvectors, Diagonalization of matrices and metric spaces,
vi) Solve partial differential equations and its application to physical problems.
vii) Laplace transforms and its application in differential equations.

References :

1. D. S. Malik, John M. Mordeson and M.K. Sen, Fundamentals of Abstract Algebra, McGraw Hill, International Edition, 1997.
2. John B. Fraleigh, A first course in Abstract Algebra, Narosa
3. Joseph Gallian, Contemporary Abstract Algebra
4. S. C. Sharma, Metric Space.
5. T. Amarnath, An Elementary Course in Partial Differential Equations, Jones and Bartlell Pub., 2011.
6. L. Debnath & D. Bhatta - Integral Transforms and Their Applications.
7. E. J. Watson - Laplace Transforms and Application, Van Nostland Reinhold Co. Ltd., 1981.

Semester : IV

Course : MATCOR 15 : 50 Marks (4 CP)

Graph Theory/ Operations Research/ Fuzzy sets & Their applications

Each year, Department will offer one course from the above list of modules, subject to the availability of resources.

Graph Theory

Syllabus :

Undirected graphs, Directed graphs, Geometrical representation of graphs, Handshaking lemma due to Euler and some basic properties of a graph. In - degree and out - degree of a vertex in a digraph. Simple digraph and underlying graph. Representation of binary relations on finite sets by digraphs. Reflexive, symmetric and transitive digraphs.

Sub graph, spanning sub graph, induced sub graph on a vertex set and induced sub graph on an edge set. Isomorphism of graphs. Walks, paths, circuits and cycles with their properties, concatenation of two walks.

Connected and disconnected graphs. A necessary and sufficient condition for a graph to be disconnected. Component of a graph, decomposition of a graph into finite number of components, acyclic graph and cycle edge of a graph. Some properties of connected graphs. Complete graphs, disconnecting sets, bridge, separating sets, distance between two vertices of a graph. Complement of a graph, Self complementary graphs, Ramsey problem. Bipartite graph and its characterization, radius and center, Diameter, Degree sequence.

Eulerian and Hamiltonian graphs: Euler trials, Euler circuits, Edge traceable graphs, Euler graphs, Euler's Theorem. Fleury's algorithm, Konigsberg bridge problem. Hamiltonian path, Hamiltonian cycle, Hamiltonian graph. A necessary condition for the existence of a Hamiltonian cycle in a connected graph. Sufficient condition for a simple connected graph to be Hamiltonian. Dirac's Theorem, Ore's Theorem and its use.

Trees and forests with their properties. Minimally connected graphs, spanning trees. weighted graphs, weight of a spanning tree and minimal spanning trees, Kruskal's algorithm for a minimal spanning tree. The shortest path problem, traveling salesman problem.

Matrix representation of graphs, adjacency matrices of graphs and digraphs and their properties, path matrix, incidence matrices of graphs and digraphs and their properties.

Cut vertices and cut edges, Vertex and edge connectivities, Blocks, Clique Number, Independence number, Matching number.

Chromatic number, Chromatic polynomial, edge colouring number, planar graphs, Kuratowski's two graphs, the Euler polyhedron formula, Euler identity for connected planar graphs, detection of planarity, Statem ent of Kuratowski Theorem, Isomorphism properties of graphs, 5 colour theorem. Statement of 4 colour theorem, Dual of a planar Graph.

Course Outcomes : After the course the student will have a strong background of graph theory. The students will be able to apply principles and concepts of graph theory in practical situations such as computer science, physical and engineering sciences.

References :

1. N. Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India, 2000.
2. C.L.Liu, Elements of Discrete Mathematics, McGraw-Hill Book Co.
3. J.P.Tremblay & R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, Mcgrow-Hill Book Co., 1997.
4. F. Harary, Graph Theory, Addison-Wesley Publishing Company, 1972.
5. J. Gross & J. Yellen, Graph Theory and its Applications, CRC Press(USA), 1999.

Operations Research

Syllabus :

Revised simplex method, Dual simplex method, Post optimal analysis.

Nonlinear programming : Karush-Kuhn-Tucker necessary and sufficient conditions of optimality, Convex programming. Quadratic programming, Wolfe's method, Beale's method.

Dynamic programming : Bellman's principle of optimality, Recursive relations, System with more than one constraint, Solution of LPP using dynamic Programming.

Inter programming : Gomory's cutting plane method , Branch and bound method.

Sequencing Models : The mathematical aspects of Job sequencing and processing problems, Processing n jobs through Two machines, processing n jobs through m machines.

Inventory control : Concept of EOQ, Problem of EOQ with finite rate of replenishment, Problem of EOQ with shortages, Multi-item deterministic problem, Probabilistic invontory models.

Queueing Theory : Basic features of queueing systems, operating characteristics of a queueing system, arrival and departure (birth & death) distributions, inter-arrival and service times distributions, transient, steady state conditions in queueing process. Poisson queueing models-M/M/1, M/M/C for finite and infinite queue length.

Course Outcomes: After completing this course, the student will be able to :

- solve nonlinear programming problems using Lagrange multiplier, Kuhn-Tucker conditions, Wolfe's and Beale's method,
- find optimal solution of dynamic programming problem,
- learn theory of sequencing models and inventory control and their applications,
- understand Queueing Theory and its applications,
- identify and formulate some real life problems into nonlinear programming problem.

References :

1. H. A. Taha - Operations Research-An Introduction. Macmillan Pub. Co., Inc., New York.
2. G. Hadley -Nonlinear and Dynamic Programming, Addition-Wesley.
3. S. S. Rao - Optimization Theory and Application, Wiley Eastern.
4. K Sarup, P. K. Gupta and Man Mohan - Operation Research, Sultan Chand & Sons.
5. J. K. Sharma - Operation Research, Mcmillan, India.
6. F. S. Hillier and G. J. Lieberman- Introduction to Operations Research, TMH, 2008
7. S. D. Sharma-Operation Research, Kedarnath & Ramnath, Meerat.
8. O. L. Mangasarian-Non linear Programming, McGraw Hill.
9. R. Panneerselvam - Operations Research, PHI, 2009.

Fuzzy sets & Their applications

Unit: 1

Fuzzy sets-Basic definitions. Level sets, Convex fuzzy sets. Basic operations on fuzzy sets.Types of fuzzy sets. Cartesian products. Algebraic products bounded sum and difference f norms and t-co norms.

Unit: 2

The Extension principle-The Zadeh's extension principle image and inverse image of fuzzy sets

Unit: 3

Fuzzy numbers. Elements of fuzzy arithmetic., Fuzzy Relations and Fuzzy Graphs-fuzzyrelations on fuzzy sets. Composition of fuzzy relations, Min-Max composition, and itsproperties.

Unit: 4

Fuzzy compatibility relations Fuzzy relation equations. Fuzzy graphs. Similarity relation.

Unit : 5

Fuzzy Logic- An overview of classical logics. Multivalued logics. Fuzzy. Propositions.
Fuzzy quantifiers. Linguistic variables and hedges.

Unit: 6

Possibility Theory-Fuzzy measures. Evidence theory, Necessity; measure. Possibility theory versus probability theory.

Unit : 7

Decision Making in Fuzzy Environment -individual decision-making. Multiperson decision making. Multicriteria decision-making. Multistage decision making fuzzy ranking methods.Fuzzy linear programming.

Course Outcomes: After completing this course, the student will be able to:

- understand basic knowledge of Fuzzy sets and Fuzzy logic,
- apply basic Fuzzy inference and approximate reasoning,
- apply basic Fuzzy system modeling methods.

References :

1. George J Klir and Tina A Folger, Fuzzy sets-Uncertainty and Information, Prentice Hall of India, 1988.
2. H. J. Zimmerman, Fuzzy Set theory and its Applications, 4th Edition, Kluwer Academic Publishers, 2001.
3. George J Klir and Bo Yuan, Fuzzy sets and Fuzzy logic: Theory and Applications, Prentice Hall of India, 1997.
4. Timothy J Ross, Fuzzy Logic with Engineering Applications, McGraw Hill International Editions, 1997.
5. Hung T Nguyen and Elbert A Walker: A First Course in Fuzzy Logic, 2nd Edition Chapman & Hall/CRC 1999.
6. Jerry M Mendel, Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions, PH PTR, 2000.
7. John Yen and Reza Langari, Fuzzy Logic: Intelligence, Control and Information, Pearson Education, 1999.

<u>**Courses : MATDSE 02, MATDSE 03 and MATDSE 04**</u>

(Advanced Paper 1**, Advanced Paper 2** and Advanced Paper 3**)

Pure Stream

1. Advanced Topology I

Syllabus :

Nets and Filters : Inadequacy of sequences. Nets & filters. Topology and convergence of nets & filters. Subnets. Ultranets & Ultra filters. Canonical way of converting nets to filters and vice-versa. Chracterizations of compactness and continuity and adherent point in terms of nets and filters. Convergence of nets and filters in product spaces.

Local Compactness and One Point Compactification.

Stone- Cech Compactification. Extension property of Stone- Cech Compactification βX . Cardinality of βN .

Embedding and Metrization. Embedding Lemma and Tychonoff Embedding. The Urysohn Metrization Theorem. The Nagata – Smirnov Metrzation Theorem (statement only).

Paracompacntess : Different types of refinements and their relationships. Paracompacntess – definition in terms of locally finite refinement, various characterizations of Paracompactness in regular spaces. A. H. Stone's Theorem concerning paracompactness of metric spaces. Partition of unity and Paracompacntess. Properties of Paracompacntess w.r.to subspaces and products.

Uniform spaces : Definition and examples. Base and subbase of a uniformity . Uniform topology, uniform continuity and product uniformity. Uniformization of topological spaces. Uniform property. Uniformity generated by a family of pseuometrics. Cauchy filter. Completeness of uniform spaces. Completion of uniform spaces. Compactness and uniformity. Uniform cover.

If time Permits:
Inductive and projective limits: Inductive and projective limits of topological spaces.
Function spaces.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge on the following :

i) Inadequacy of sequences, Nets and Filters , Chracterizations of compactness and continuity and adherent point in terms of nets and filters,
ii) Local Compactness and One Point Compactification, Stone- Cech Compactification, Extension property of βX and Cardinality of βN,

iii) The Urysohn Metrization Theorem. The Nagata – Smirnov Metrzation Theorem,
iv) Paracompacntess, Partition of unity, A. H. Stone's Theorem,
v) Uniform spaces and Uniform topology, uniform continuity and product uniformity, Uniformity generated by a family of pseuometrics, Completion of uniform spaces,
vi) Inductive and projective limits, Function spaces.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Advanced Topology I, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. General Topology by J. L. Kelley, Van Nostrand.
2. General Topology by S. Willard, Addison-Wesley.
3. Topology by J. Dugundji, Allyn and Bacon.
4. Topology, A first course by J. Munkres, Prentice Hall, India.
5. Introduction to topology and modern analysis by G. F. Simmons, McGraw Hill.
6. Introduction to General topology by K. D. Joshi, Wiley Eastern Ltd.
7. General Topology by Engelking, Polish Scientific Publishers, Warszawa.
8. Counter examples in Topology by L. Steen and J. Seebach.

2. Advanced Topology II

Syllabus :

Algebraic Topology :

Covering spaces and covering maps – properties and examples, Path Lifting and Monodromy theorems, Van Kampen's theorem (with a discussion of free and amalgamated products of groups), computing fundamental groups via covering spaces.

Homology - Homology: simplicial homology; singular homology; the Mayer-Vietoris sequence; The Jordan-Brouwer Separation Theorem; the Universal Coefficient Theorem; the Kunneth Formula; CW complexes; cellular homology and computations for projective spaces; the Lefschetz Fixed Point Theorem.

Rings of Continuous Functions :

The ring C(X) & its subring $C^*(X)$, their Lattice structure. Ring homomorphism and lattice homomorphism.
Zero-sets, cozero-sets, completely separated sets and its characterization, C-embedding & C* -embedding and their relation, Urysohn's extension theorem . Characterizations of Normal spaces and Pseudocompact spaces in terms of C-embedding & C* -embedding.

Ideals, maximal ideals, prime ideals, Z- ideals; Z-filters, Z- ultrafilters, prime filters and their relations. Convergence of Z – filters, cluster points, prime Z – filters and convergence and fixed Z-filters .

Completely regular spaces and the zero-sets, weak topologies determined by C(X) and $C^*(X)$. Stone-Čech's therem concerning adequacy of Tychonoff spaces X for investigation of C(X) and $C^*(X)$.

Fixed ideals and compactness, fixed maximal ideals of C (X) and $C^*(X)$, their characterizations.
Structure spaces.

If time permits :

Topological groups: Basic properties of topological groups, separation properties, subgroups, quotient groups and connected groups.

Course Outcomes: On completion of this course , the students will be able to identify , analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Covering spaces and covering maps,Path lifting property and Homotopy lifting,
ii) Monodromy theorems, Deck transformation, Van Kampen's theorem,
iii) Singular Homology, Mayer-Vietoris sequence, Idea of Cohomology,
iv) C-embedding & C^* -embedding and their relation, Urysohn's extension theorem,
v) maximal ideals, prime ideals, Z- ideals; Z-filters, Z- ultrafilters,
vi) fixed maximal ideals of C (X) and $C^*(X)$, their characterizations,Structure spaces.

vii) Topological groups.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Advanced Topology II, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. W. S. Massey – Algebraic Topology.
2. W. S. Massey – Singular Homology Theory.
3. E. H. Spanier – Algebraic Topology.
4. B. Gray – Homotopy Theory An Introduction to Algebraic Topology.
5. C. R. Bredon – Geometry and Topology.
6. Richard E. Chandler, Hausdorff Compactifications (Marcel Dekker, Inc. 1976).
7. L. Gillmen and M. Jerison, Rings of Continuous Functions (Von Nostrand, 1960).
8. Topological Structures (Halt Reinhurt and Winston, 1966).

3. Advanced Functional Analysis

Syllabus :

Convex sets, convex hull, Representation Theorem for convex hull. Symmetric sets, balanced sets, absorbing sets and their properties, absolutely convex sets. Topological vector spaces, homeomorphisms, local base, locally convex topological vector spaces, bounded sets, totally bounded sets, connectedness and their basic properties. Separation properties of a topological vector space, compact and locally compact topological vector space and its properties on finite dimensional topological vector spaces, convergence of filter, completeness, Frechet space, quotient spaces, separation property by hyperplane on locally convex topological vector spaces, Linear operators over topological vector space, Boundedness and continuity of linear operators, Minkowski functionals and its basic properties, Hyperplanes, Separation of convex sets by Hyperplanes, Extreme points, Krein-Milman Theorem on extreme points, Metrizability of topological vector spaces.

Geometric form of Hahn Banach Theorem. Uniform boundedness principle, open mapping theorem and closed graph theorem for Frechet spaces. Banach-Alaoglu theorem.

Seminorms and its properties, Generating family of seminorms in locally convex topological vector spaces, Criterion for normability of a topological vector space (Kolmogorov Theorem).

Weierstrass Approximation Theorem in C[a,b], best approximation theory in normed linear spaces, uniqueness criterion for best approximation. Separable Hilbert Space, Strict convexity and uniform convexity of a Banach space with examples, Uniform approximation, Haar condition, Haar uniqueness theorem.

Only statements of Clarkson's Renorming Lemma and Milman and Pettit's theorem, Uniform convexity of a Hilbert space, Reflexivity of a uniformly convex Banch space.

Course Outcomes: Upon successful completion, students will have the knowledge and skills to explain the fundamental concepts of functional analysis and their role in modern mathematics and applied contexts. Moreover, students will be able to demonstrate accurate and efficient use of functional analysis techniques.

References :

1. W. Rudin, *Functional Analysis*, TMG Publishing Co. Ltd., New Delhi, 1973**.**
2. E. Kreyszig, *Introductory Functional Analysis with Applications,* Wiley Eastern, 1989.
3. G. Bachman and L. Narici, *Functional Analysis,* Academic Press, 1966.
4. A. E. Taylor- *Functional Analysis*, John Wiley and Sons, New York, 1958.
5. L. Narici & E. Beckenstein, *Topological Vector spaces*, Marcel Dekker Inc, New York and Basel, 1985.
6. A. A. Schaffer, *Topological Vector Spaces*, Springer, 2nd Edn., 1991.
7. J. Horvoth, *Topological Vector spaces and Distributions*, Addison-Wesley Publishing Co., 1966.

4. Algebraic Topology

Syllabus :

Homotopy and Homotopy classes. Homotopy equivalences, Null homotopy, Relative homotopy, Composite of homotopic spaces.

Contractible spaces, deformation, strong deformation retraction, Path-connected spaces - their union, intersection and continuous images.
Product and inverse of paths. Homotopy of paths and products of homotopic paths.

Covering spaces and covering maps. Properties of covering maps. Path lifting property and Homotopy lifting theorem.

Fundamental group : Definition and verification. Homomorphism and isomorphism of fundamental groups. Fundamental groups of Circle. Fundamental groups of some known surfaces - Cylinder, punctured plane, Torus, etc.

Finite Simplicial Complexes : Simplicial complexes. Polyhedra and Triangulation.
Simplicial approximation, barycentric subdivision and simplicial approximation theorem.

Simplicial Homology : Orientation of simplicial complexes. Simplicial chain complexes, boundaries and cycles, homology groups – some examples. Induced homomorphisms. Reduced homology groups.
Some applications, e.g., Invariance of dimension, no-retraction theorem, Brower's fixed point theorem, etc.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Homotopy, Contractible spaces, deformation, strong deformation retraction,
ii) Covering spaces and covering maps,Path lifting property and Homotopy lifting,
iii) Fundamental groups of Circle, Cylinder, punctured plane, Torus, etc.,
iv) Simplicial complexes. Polyhedra and Triangulation, barycentric subdivision and simplicial approximation theorem,
v) Simplicial Homology, homology groups, no-retraction theorem, Brower's fixed point theorem.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of algebraic topology to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. Algebraic Topology, A. Hatcher, Cambridge University Press.
2. Algebraic Topology, W.S. Massey, Springer (GTM).
3. Algebraic Topology : A first course, M.J. Greenberg and J. R. Harper, Perseus books, Cambridge.
4. Algebraic Topology : A Primer, S. Deo, Hindustan Book Agency (trim 27).
5. Categories for the Working Mathematicians (second edition), S. MacLane, Springer (GTM).
6. Algebraic Topology : A First Course, Springer-Verlag,1995.
7. Essential Topology, Springer ,Martin D. Crossley.
8. Algebraic Topology, J . Munkres.

5. Advanced Real Analysis

Syllabus :

Representation of real numbers by series of radix fractions. Sets of real numbers,Derivatives of a set. Points of condensation of a set. Structure of a bounded closed set.

Perfect sets. Perfect kernel of a closed set. Cantor's nondense perfect set. Sets of first and second categories, residual sets. Baire one functions and their basic properties. One-sided upper and lower limits ofa function. Semicontinuous functions. Dini derivates of a function. Zygmund's monotonicity criterion.

Vitali's covering theorem. Differentiability of monotone functions and of functions of bounded variation. Absolutely continuous functions, Lusin's condition ,characterization of AC functions in terms of VB functions and Lusin's condition. Concepts of VB*, AC*,VBG*, ACG* etc. functions. Characterization of indefinite Lebesgue integral as an absolutely continuous function.

Generalized Integrals **:** Gauge function. Cousin's lemma. Role of gauge functionin elementary real analysis. Definition of the Henstock integral and its fundamental

properties. Reconstruction of primitive function. Cauchy criterion for Henstock integrability. Saks-Henstock Lemma. The Absolute Henstock Integral. The McShane integral. Equivalence of the McShane integral, the absolute Henstock integral and the Lebesgue integral. Monotone and Dominated convergence theorems. The Controlledconvergence theorem.

Definition and elementary properties of the Perron integral and its equivalence with the Henstock integral. Definition of the (special) Denjoy integral and its equivalence with the Henstock integral (characterization of indefinite Henstock integral as a continuous ACG*function). Density of arbitrary sets. Approximate continuity. Approximate derivative.

Course Outcomes: After completing the course, the students should be able to recognize, understand and apply concepts and methods in advanced real analysis. Also, they will be able to apply the acquired knowledge in signals and Systems, Digital Signal Processing etc. and conduct researches on high international level in advanced real analysis.

References :

1. E. W. Hobson **:** The Theory of Functions of a Real Variable (Vol. I and II).

2. I. P. Natanson **:** Theory of Functions of a Real Variable (Vol. I and II).

3. R. Henstock **:** Lectures on the Theory of Integration.

4. E. J. McShane **:** Unified Integration.

5. S. Saks **:** Theory of the Integral.

6. Advanced Complex Analysis

Syllabus :

Elementary properties of holomorphic functions: Basic properties of holomorphic functions, relations with the fundamental group and covering spaces; the Open Mapping Theorem;

The Maximum Modulus Principle : The Schwarz Lemma, The Phragmen- Lindeloff Method, a converse of Maximum Modulus Theorem.

Approximation by Rational functions: Runge's Theorem, simply connected regions, the Mittag-Leffler's theorem for Meromorphic function.

Zeros of holomorphic functions: Infinite products, the Weierstrass Factorization Theorem, Jensen's formula, The Muntz-Szasz theorem.

Analytic Continuation : Direct analytic continuations, uniqueness of analytic continuation along a curve, Monodromy theorem and its consequence, the Little Picard Theorem.

Conformal mapping : Normal families, the Riemann mapping Theorem.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Basic properties of holomorphic functions,
ii) The Phragmen- Lindeloff Method, a converse of Maximum Modulus Theorem,
iii) the Mittag-Leffler's theorem for Meromorphic function,
iv) the Weierstrass Factorization Theorem, Jensen's formula, The Muntz-Szasz theorem,
v) Monodromy theorem and its consequence, the Little Picard Theorem,
vi) the Riemann mapping Theorem,
vii) multilinear algebra, differential forms, the Lie derivative..

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Advanced Complex Analysis, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions.

References:

1. J.B. Conway, Functions of one complex variables Springer - Verlag, International student Edition, Narosa Publishing Co.
2. L. Hahn, B. Epstein, Classical Complex Analysis, Jones and Bartlett, India, New Delhi, 2011
3. W. Rudin, Functional analysis.
4. S. Lang, Real analysis.
5. L.V. Ahlfors, Complex Analysis, Mc. Graw Hill Co., New York, 1988.
6. W. Rudin, Real and complex analysis, McGraw-Hill, 1987.

7. Harmonic Analysis

Syllabus :

Fourier analysis : Fourier series, pointwise and uniform converges of Fourier series, Fourier transforms, Riemann-Lebesgue lemma, inversion theorem, Parseval identity.
Topological groups: Definition, Basic properties, subgroups, quotient groups, locally compact topological groups, examples. Compact groups: Representations of compact groups, Peter-Weyl theorem, Examples such as SU(2) and SO(3).

Positive Borel measure, Riesz representation theorem, regularity properties of Borel measures.

Haar measure and Haar integral: Invariant measure and Integration, existence and uniqueness of Haar measure and Haar integral on locally compact topological group, Examples of Haar measures Haar Integration.

Elements of Banach algebras: Banach algebra, examples of Banach algebra, algebra with involution, Analytic properties of functions from C to Banach algebras, spectrum and its compactness, commutative Banach algebras, Maximal ideal space, Gelfand topology, Gelfand representation theorem.

Generalization of Fourier transform : Fourier transform on L 0 (G) and L(G) (G being a locally compact topological group) Positive definite functions, Bochner characterization, inversion formula, Plancheral theorem, Pontrjagin Duality theorem.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Fourier series, convergence of Fourier series, Riemann-Lebesgue lemma
ii) Basics of Topological groups,
iii) Haar measure and Haar integral,
iv) Banach Algebra and Gelfand topology,
v) Fourier transform on locally compact topological groups,
vi) Plancheral theorem, Pontrjagin Duality theorem.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Harmonic Analysis, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions.

References :

1. Hewitt and Ross; Abstract Harmonic analysis (Vol. I & II); Springer-Verlag, 1963.
2. Bachman, Narici and Beckenstein; Fourier and Wavelet Analysis; Springer.
3. Folland, G. B., A Course in Abstract Harmonic Analysis, CRC Press, 1995.
4. Deitmar, Anton, A First Course in Harmonic Analysis, second edition, Springer, 2002.
5. Walter Rudin; Real and Complex Analysis; McGraw-Hill Book Company, 1921.
6. Katznelson, Yitzhak, An Introduction to Harmonic Analysis, third edition, CUP, 2002.
7. Helson, H., Harmonic Analysis, Addison Wesley, 1983.
8. de Vito, C., Harmonic Analysis - A Gentle Introduction, Jones & Bartlett, 2007.
9. R. R. Goldberg; Fourier Transforms; Cambridge, N.Y., 1961.

8. Commutative Algebra

Syllabus :

Properties of Maximal, prime and primary ideals, radical, nil-radical and Jacobson radical, local ring, Nakayama’s lemma, prime spectrum of a ring and Zariski topology, Noetherian and Artinian rings, Hilbert's Nullstellensatz theorem. Finitely generated modules, tensor product of modules, exactness properties of tensor product.

Rings and Modules of fractions, localization and local properties, primary decomposition and associated primes, Integral dependence and independence, integral closure, integrally closed integral domain, Going up and going-down theorems.

Valuation rings, discrete valuation ring, Dedekind’s domain, graded rings and modules, completion of modules, Krull intersection theorem. Dimension theory – Dimension theorem of Noetherian local rings.

Course Outcomes: On successful completion of this course, students will be able to apply its methods in related subjects of Mathematics. Moreover, they should be able to participate in scientific discussions and begin with own research in commutative algebra.

References :

1. M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, Addison–Wesley, 1969.
2. N. Bourbaki, *Commutative Algebra*, Hermann, 1972.
3. D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer–Verlag, 1995.
4. I. Kaplansky, *Commutative Rings*, The University of Chicago Press, 1974.
5. E. Kunz, *Introduction to Commutative Algebra and Algebraic Geometry*, Birkh¨auser, 1985.
6. H. Matsumura, *Commutative Algebra*, Benjamin, 1970.
7. H. Matsumura, *Commutative Ring Theory*, Cambridge University Press, 1986.
8. M. Nagata, *Local Rings*, Wiley Interscience, New York, 1962.
9. O. Zariski and P. Samuel, *Commutative Algebra*, Vol. 1, Van Nostrand, 1958.

Applied Stream

1. Quantum Mechanics

Syllabus :

Experimental background of quantum mechanics; deBroglie waves, Wave-particle duality; Wave functions and Schrodinger equation; Uncertainty relation.

Statistical interpretation of wave functions, expectation values, Ehrenfest's theorem; Time-independent Schrodinger equation; Energy eigenfunction : Discrete and continuous energy eigenvalues; Infinite and finite square well problems: Parity, Simple harmonic oscillator: Algebraic and analytic methods of solution, Dirac delta function potential, free particle: wave packets.

Representation of observables, Dirac's bra-ket notations, mathematical set up on Hilbert space. Equations of motion: Schrodinger picture, Heisenberg picture, Interaction picture.

The Hydrogen atom, angular momentum, spin. Rotation, angular momentum and unitary groups, Generators of U(n) and SU(n), representation in terms of coordinate and momenta. Clebsch-Gordan coefficients, Wigner-Eckart theorem. Space inversion, time reversal. O(4) symmetry of Hydrogen atom.

Identical particles, Bosons, Fermions; Pauli exclusion principle; Solids: Free electron gas, Band structure. Quantum statistical mechanics: Maxwell-Boltzmann, Fermi-Dirac and Bose-Einstein distributions. Blackbody spectrum.

First and second order perturbations, degenerate perturbation theory. Fine structure of Hydrogen, spin-orbit coupling, Zeeman effect.

Variational method: Rayleigh-Ritz variational principle; Hydrogen molecule ion, ground state of helium atom.

Relativistic quantum mechanics: Klein-Gordon equation, plane wave solution. Dirac equation, covariant form, charged particle in electromagnetic field, equation of continuity. Dirac hole theory. Spin of the Dirac particle.

Course Outcomes: At the end of this course a student should be able to :

- Understand the fundamentals of quantum mechanics,
- Create better grasp on different branches of mathematical physics,
- Provide an opportunity to recapitulate application of higher pure mathematics,
- Open the gateway to modern electronics and nano science.

References :

1. P.A.M. Dirac, The Principles of Quantum Mechanics, Oxford University Press.
2. D. J. Griffiths, Introduction to Quantum Mechanics, Pearson Prentics Hall.
3. B.H. Bransden and C.J. Joachain, Introduction to Quantum Mechanics.
4. L. I. Schiff, Quantum Mechanics, McGraw-Hill, New York, 1968.
5. R. Eisberg, R. Resnick, Quantum Physics of Atoms, Molecule, Solids, Nuclei and Particles, Wiley.
6. A. Das, Lectures on Quantum Mechanics, Hindusthan Book Agency, New Delhi, 2003.
7. E. Merzbacher, Quantum Mechanics,Wiley, New York.
8. C. Cohen-Tannoudji, B. Diu, and F. Laloe, Quantum Mechanics , Wiley- Interscience Publication.
9. R.P. Feynman, The Feynman Lectures on Physics, 3 Vols., Narosa Publ., New Delhi.
10. L.E. Ballentine, Quantum Mechanics, World Sci. Publ., Singapore.
11. T.F.Jordan, Quantum Mechanics in Simple Matrix Form, Dover Publ.
12. M. Chester, Primer of Quantum Mechanics, Dover Publ.
13. J.P. McEvoy and O. Zarate, Introducing Quantum Theory, Icon Books UK, Singapore.

2. Plasma Dynamics

Syllabus :

Basic properties of plasmas :

Definition of Plasma as an ionized gas. Thermal ionization, Saha equation, Basic defining properties of plasma, Debye shielding, Plasma parameters, plasma frequency, Collisions. Natural occurrence of Plasma. Applications of plasma physics.

Motion of Charged Particle:

Motion of charged particles in electric and magnetic fields: Larmor orbits, Particle drifts: $\mathbf{E}\times\mathbf{B}$ drift, polarization drift, curvature drifts, grad B drifts. Magnetic moments, Adiabatic invariants. Concept of Ponderomotive force. Magnetic mirror (concept of plasma confinement).

Plasma kinetic theory:

Vlasov equation: Equilibrium solutions, Electrostatic Waves. Concept of Landau damping.

Plasma Fluid Theory:

Derivation of fluid equations from the Vlasov equation. Plasma oscillations, Langmuir waves, Dielectric Function, ion-acoustic waves, Electromagnetic waves. Upper and lower hybrid waves, Alfven waves, Ion and Electron cyclotron waves.

Nonlinear Plasma Theory:

Concept of nonlinearity and dispersion. Korteweg-de Vries equation: ion acoustic solitary wave and its solution. Nonlinear Schrödinger equation and Envelope soliton.

Course Outcomes: At the end of this course a student should be able to :

- understand collective nature of plasma dynamics by developing concepts of Debye screening collective behavior and quasi neutrality,
- describe motion of charged particles in electric and magnetic fields,
- derive the basic set of fluid equations to study plasma properties,
- know the concept of Landau damping,
- describe the propagation of waves in plasmas and understand the concept of nonlinearity and dispersion relation.

References :

1. F. F. Chen - Introduction to Plasma Physics, Plenum Press, New York and London (1977).
2. D. R. Nicholson - Introduction to Plasma Theory, John Wilely and Sons, NY, (1983).
3. T. J. M. Boyd and J. J. Sanderson - The Physics of Plasmas, Cambridge University Press, (2003).
5. R. C. Davidson - Methods in Nonlinear Plasma Theory, Academic Press, New York and London (1972).
6. J. A. Bittencourt - Fundamentals of Plasma Physics, Springer-Verlag New York, (2008).
7. N. A Krall and A. W. Trievelpiece - Principles of Plasma Physics, McGraw Hill Kogakusha, Ltd., Tokyo, New Delhi etc. (1973).
8. P. C. Clemmow and J. P. Dougherty - Electrodynamics of Particles and Plasma.
9. B. Chakraborty - Principles of Plasma Mechanics.

3. Theory of Waves in Solids

Syllabus :

1. Elastodynamic theory. Linearized equations of Elasticity. Uniqueness of solutions. Scalar and vector potentials. Wave motion generated by body forces. Point loads. Boundary value problems.
2. Half-space subjected to uniform surface traction. Waves in one-dimensional stress. Harmonic waves.
3. Elastic waves in unbounded medium. Plane waves. Two dimensional wave motion with axial symmetry. Propagation of wave fronts.
4. Plane harmonic waves in elastic half-spaces. P,Sh and SV waves. Reflection. Rayleigh and Stoneley waves.
5. Elastic waves in waveguides.SH waves in a layer.SH modes. Energy Transport . Group velocity and dispersion. Love waves. Lamb waves.
6. Waves in rods and shells. Thin rods. Finite rods. Frequency equation for solid circular rods, Torsional, flexural and longitudinal modes, waves in cylindrical shells.

Course Outcomes:

1. On completion of the course, students will be conversant with propagation of waves in rods, plates and half-spaces.
2. They will be introduced to the basic seismological waves and acoustic waves.
3. The course will be beneficial to students interested in research in applied mechanics or geophysics.

References :

1. Elastodynamics, Volume II, A.C. Eringen and E, S. Suhubi, Academic Press, 1974.

2. Wave Propagation in Elastic Solids , J. D. Achenbach, North –Holland, 1975.

3. Wave motion in elastic solids, Karl F Graff , Dover, 1975.

4. Elastic Waves in layered media, W. M. Ewing, W. S. Jardetzky and F Press, McGraw Hill, 1957.

4. Advanced Dynamical Systems and Chaotic Dynamics

Syllabus :

Nonlinear Systems. Bifurcations and Symmetry breaking. – the origin of Bifurcation Theory. Examples of different types of bifurcations. Transcritical, pitchfork, saddle-node. Centre manifolds. Bifurcation of equilibrium solutions and Hopf bifurcation.

Difference equations.The logistic map. Periodic solutions and their stability.

Introduction to the theory of Chaos. The Lorenz equations and associated maps. Duffing's equation with negative stiffness.One dimensional chaos. The quadratic map. The tent map.Strange attractors.

Bifurcations in one dimensional maps. Period doubling bifurcations. The Feigenbaum number.Two dimensional maps. Bifurcation in two dimensional maps.

Cantor sets. Dimension and fractals.

Hamiltonian systems.Recurrence. Periodic solutions. Invariant torus and chaos.

Course Outcomes: On completion of this course the students would be able to :

1. apply the ideas of dynamical systems theory to understand and explain various complex phenomena of physics and biology,

2. pursue research in complex dynamical systems, mathematical biology , fractal set theory and other related fields.

References :

1. Dynamical systems differential equations, maps and chaotic behavior, D K Arrowsmith and C M Place., Chapman and Hall.
2. Chaotic Dynamics, Baker and Gollub.
3. Nonlinear Systems, P.G. Drazin, CUP 1992.
4. Nonlinear Differential equations and Dynamical Systems, Verhulst.
5. Nonlinear Oscillations, Dynamical systems and bifurcations of vector fields .J Guckenheimer, P Holmes. Springer NY, 1983.
6. Nonlinear Dynamics and Chaos, S.H. Strogatz, Perseus Books, USA, 1994.
7. Differential equations, Dynamical systems and an introduction to chaos, M.W.Hirsch, S.Smale, R.L. Devaney , Academic Press, 2004.
8. Chaos: An Introduction to Dynamical Systems, Kathleen T. Alligood, James A. Yorke, and Tim Sauer Springer NY, 1997.

5. Solid Mechanics

Syllabus :

Formulation of Problems in Elasticity: Review of field equations. Boundary conditions and fundamental problem classifications. Stress and displacement formulation. ‘Uniqueness of solutions. Clapeyron’s Theorem. Saint-Venants principle.

Problems in Elastostatics: Plane deformation. Plane stress. Boundary conditions. Airy’s stress function. Biharmonic boundary value problems. The first and second boundary value problems. Existence and uniqueness of solutions. Conformal maps. Simply connected domains. Solution of basic problem in a circular region.

Extension, Torsion and Flexure of Beams : Statement of Problem. Extension by longitudinal forces. Beam stretched by its own weight. Bending by terminal couples. Torsion of circular shaft. Torsion of cylindrical bars. Torsion function. Neumann’s problem. Stress function. Dirichlet’s problem. Flexure of beams by terminal loads. Neumann and Dirichlet’s problems. Centre of flexure. Bending by a load along the principal axis. Bending of rectangular beams.

Problems in Elastodynamics: Uniqueness of solutions. Wave propagation in infinite region. Vector and scalar potentials. Half-space. Rayleigh waves.

Course Outcomes:

- This course is intended to give the students an introduction to different types of problems arising in the Theory of linear Elasticity.
- On completion of this course students will have learnt the fundamental concepts required for research in Applied Mechanics or Geophysics.

References :

1. I. S. Sokolnikoff: Mathematical Theory of Elasticity. McGraw Hill, 1956.
2. 2. A. E. H. Love: A treatise on mathematical theory of elasticity. Dover, 1954.
3. P.L. Gould: Introduction to linear elasticity. Springer-Verlog, 1994.
4. N. I. Muskhelisvili: Some basic problems on the theory of elasticity. Nordhoff, 1953.
5. Y. C. Fung: Foundation of solid mechanics. Prentice Hall, 1965.
6. L. D. Landau and E. M. Lifshitz: Theory of Elasticity. Pergamon Press, 1989.
7. S. Timoshenko and S. N. Goodier: Theory of Elasticity. McGraw Hill, 1970.
8. V. Z. Parton and P. I. Perlin: Mathematical Methods of the theory of elasticity. vol. I, II, Mir Publishers, 1984.
9. Elastodynamics, Volume II, A. C. Eringen and E. S. Suhubi, Academic Press, 1970.

6. Mathematical Biology

Syllabus :

A. Mathematical Models of Population Biology or Ecology

1. Deterministic models. Continuous growth models. Logistic growth law. Allee effect. Bacterial growth. Harvesting. Functional responses. The spruce budworm population.

Models of interacting populations. The Lotka-Volterra model for competition. Competition between n species. The Lotka Volterra predator –prey model.Complexity and stability in a generalised predator-prey system.Predator-prey models with logistic growth in prey and Holling-type responses. Analysis of such models with limit cycle periodic behaviour. Mutualism.Host parasite model.

2. Stochastic processes and stochastic models. Pure birth process, Pure death process, Birth and death process. Linear birth-death-immigration-emigration processes. Effects of both immigration and emigration on the dynamics of population.

3. Biological mechanisms responsible for "time-delay". Discrete and continuous time-delay. The single species logistic model with the effect of time-delay. Stability of equilibrium position for the logistic model with general delay function. Stability of logistic model for discrete time lag. Time-delayed H-P model together with their stability analysis.

4. Spatial population models. Metapopulations. Reaction-diffusion model. Models for animal dispersal.

5. Biological waves. Single -species model. Fisher-Kolmogoroff equation and travelling wave solutions.

B. Models of Epidemics.

Introduction; Some basic definitions. Simple epidemic model, General epidemic model. Kermack-McKendrik threshold theorem. Recurring epidemic model. A comparative study of these models. Control of an epidemic. Stochastic epidemic model without removal. Models having multiple infections. Epidemic model with multiple infections. Stochastic epidemic model with removal. Stochastic epidemic model with removal, immigration and emigration. Special discussion on the stochastic epidemic model with carriers. Simple extensions of SIR model: Different case studies --- (i) Loss of immunity, (ii) Inclusion of immigration and emigration, (iii) Immunization. SIR endemic disease model.

Course Outcomes:

- After completion of this course, students should be able to formulate realistic mathematical models for diverse biological phenomena and analyse them mathematically to explain the observations as obtained from experiments, clinical trials and observations.
- Students would learn to mathematically predict the outcome in a situation by constructing and theoretically analysing a model.
- The students will learn how to develop mathematical models which provide ways to design and evaluate protocols to manage and control animal populations, natural resources like forests, wildlife, fisheries, and outbreak of diseases.

References :

1. J. D. Murray – Mathematical Biology 1. An Introduction., Springer-Verlag, Berlin (2002).

2. R. M. Andersson and R M May-- Infectious Diseases of Humans : Dynamics and control OUP, (1991).

3. J. N. Kapur --- Mathematical Models in Biology and Medicine, East West Press Pvt Ltd (1985)

4. 6. R. W. Poole --- An Introduction to Quantitative Ecology, McGraw- Hill, (1974).

5. E. C. Pielou -- An Introduction to Mathematical Ecology, Wiley, New York, (1977).

6. R. Rosen -- Foundation of Mathematical Biology (vol. I& II), Academic Press, (1972).

7. R. M. May --- Stability and Complexity in model ecosystems ,Princeton University Press, (2001).

8. Mark Kot – Elements of Mathematical Ecology, Cambridge University Press, (2003).

9. J. M. Smith-- Mathematical Ideas in Biology. CUP, (1968).

10. L. J. S. Allen-- An introduction to Mathematical Biology, Pearson/Prentice Hall, (2007).

7. Advanced Operations Research

Syllabus :

Network Analysis: Network definitions, Minimal Spanning Tree Algorithm, Shortest Route Algorithms, Max-flow Min-cut theorem, Genaralized Max-flow Min-cut theorem, linear programming interpretation of Max-flow Min-cut theorem, minimum cost flows. A brief introduction to PERT and CPM, Components of PERT/CPM Network and precedence relationships, Critical path analysis, PERT analysis in controlling project.

Optimal Control Theory: Performance criterion, Unconstrained systems, Application of calculus of variation, constrained systems, Pontryagin's principle, Quadratic performance criterion, Regulator problem.

Replacement Models: Types of replacement problems, replacement of items deteriorates with time, Replacement policy for equipments when value of money changes with constant rate during the period, Replacement of low cost items, Group replacement, Individual replacement policy, Mortality theorem, Recruitment and promotional problems.

Matrix Game: Definition of a non-cooperative game. Admissible situation and the equilibrium situation, strategic equivalence of games. Antagonistic Games, Saddle points. Matrix Games. Mixed strategies. Existence of minimaxes in mixed strategies. Convex sets. The value of the game and optimal strategies.

Continuous Games: Continuous games on unit square. Continuous game. Equilibrium Situation. Fundamental Theorem. Devices for Computing and Verifying Solutions.

Differentiable Game: Two person deterministic continuous differential games, Two person zero-sum differential games, Pursuit games, Co-ordination differential games, Noncooperative differential game.

Simulation: Basic concepts, Monte Carlo method, Random number generation, Waiting the simulation model, New process planning through simulation, Capital budgeting through simulation.

Course Outcomes: Upon completion of this course, the student will be able to**:**

- formulate operation research models to solve real life problem,
- understand the mathematical tools that are needed to solve optimization problems,
- describe Optimal Control Theory and their applications,
- analyze game theory,
- understand skills and knowledge of operations research and its application in industry.

References :

1. Joseph J. Madder, Cecil R, Philips, Project Management with PERT and CPM.
2. Panel A. Jensen, Wesley Barness J., Network flow programming, John Wiley and Sons, 1980.
3. OR methods and Problems - Sasieni Maurice, Arther Yaspan, Lawrence Friedman.
4. Elmagharby Salah E., Activity Network Project Planning and Control by Network Models, John Wiley and Sons.
5. Operations Research – H. A. Taha.
6. C. Mohan and K. Deep, Optimization Techniques, New Age Science, 2009.
7. Operations Resarch - T.L. Satty.

8. Advanced Fluid Dynamics

Syllabus :

1. Two and Three dimensional Inviscid incompressible fluid flow : : Field equations; Irrotational motion in simply connected and multiply connected regions. Source, sink,doublet. Image systems. Motion of solid bodies in fluid. Axi-symmetrical motion, Stokes' stream function, Two dimensional motion, Stream function, complex potential, motion of translation and rotation of circular and elliptic cylinders in an infinite liquid, Ciculation. Kelvin's Theorem. Cyclic and acyclic motion. Superposition of motion, circle theorem, Blasius theorem, Kutta Joukowski's theorem.

2. Surface waves, progressive waves in deep and shallow water, Stationary waves, energy and group velocity.

3. Viscous incompressible fluid flow: Similarity, Reynold's number, Flow between parallel plates. Couette and plane Poiseuille flow. Flow through pipes of circular, annular and elliptic cross sections.

4. Laminar Boundary layer.

5. Inviscid compressible flow : Field equations, Circulation, Propagation of small disturbance. Mach number and cone, Bernoulli's equation. Irrotational motion, Velocity potential. Bernoulli's equation in terms of Mach number. Pressure, density, temperature in terms of Mach number, Critical conditions. Steady channel flow, Area-velocity relation. Mass flow through a converging nozzle. Flow through a de-Laval nozzle. Normal shock waves, Governing equations and the solution.

6. Viscous compressible flow: Field equation of compressible flow, Crocco-Vazsonyl equation

Course Outcomes:

1. This course introduces fundamental ideas of fluid dynamics which can be further applied to problems of mechanical engineering.

2. On completion of this course, students would be able to enter research work in Advanced Fluid Theory and Computational Fluid Dynamics (CFD).

References :

1. H. Lamb- Hydrodynamics, Dover Publication.
2. L.M. Milne-Thomson, Theoretical Hydrodynamics.
3. L. Prandtl- Essential of Fluid Dynamics, Hafnen, Pub. Co.
4. P.K. Kundu and Iva M.Cohen-Fluid Dynamics, Har Court, India.
5. J.J. Stoker - Water waves, the mathematical theory with application, Interscience Publ.
6. S.I. Pai- Viscous Flow Theory, Princeton.
7. F. Chorlton- Text Book of Fluid Dynamics, CBS Publ.
8. S.W.Yuan - Foundations of fluid Mechanics, PHI India, 1969.