SYLLABUS FOR M. SC.
IN
MATHEMATICS

Based on Two Year NEP PG Curriculum

Effective from Academic Session 2026-2027

Department of Mathematics
West Bengal State University
Berunanpukuria, P.O. - Malikapur
Barasat, Kolkata -700 126
West Bengal, India

Department of Mathematics

Semester wise Course Structure for Two year NEP PG Curriculum

Semester	Type of course	Paper Name	Credit	Marks	Total
I	MTM2P COR 01T	Algebra	4	50	**Marks : 300** **Credits : 22**
	MTM2P COR 02T	Linear Algebra	4	50	
	MTM2P COR 03T	Real Analysis	4	50	
	MTM2P COR 04T	Gr. A: Graph Theory Gr. B: Special and Generalized Functions	2+2=4	25+25=50	
	MTM2P COR 05T	Classical Mechanics	4	50	
	MTM2P AEC 01M	Computer Aided Numerical Analysis with Python	2	50	
II	MTM2P COR 06T	Topology	4	50	**Marks : 250** **Credits : 20**
	MTM2P COR 07T	Functional Analysis	4	50	
	MTM2P COR 08T	Electromagnetic Theory and Special Theory of Relativity	4	50	
	MTM2P COR 09T	Integral Equations and Calculus of Variations	4	50	
	MTM2P DSE 01T	Elective Paper I*	4	50	
III	MTM2P COR 10T	Measure and Integration	4	50	**Marks : 300** **Credits : 22**
	MTM2P COR 11T	Operator Theory and Banach Algebra	4	50	
	MTM2P COR 12T	Nonlinear Differential Equations and Dynamical Systems	4	50	
	MTM2P COR 13T	Advanced Numerical Analysis and PDE	4	50	
	MTM2P DSE 02T	Elective Paper II*	4	50	
	MTM2P SEC 01M	Mathematical Computing with Mathematica/Matlab & Introduction to LATEX	2	50	
IV	MTM2P DSE 03T	Advanced Paper 1**	4	50	**Marks : 300** **Credits : 24**
	MTM2P DSE 04T	Advanced Paper 2**	4	50	
	MTM2P DSE 05T	Advanced Paper 3**	4	50	
	MTM2P DSE 06T	Advanced Paper 4**	4	50	
	MTM2P COR 14M	Project/ Dissertation	4	50	
	MTM2P COR 15M	Seminar	4	50	

* List of Elective Paper I :

One topic has to be chosen by a candidate from the following (subject to the availability of resources):

1. Continuum Mechanics I
2. Differential Manifold
3. Operations Research

* List of Elective Paper II :

One topic has to be chosen by a candidate from the following (subject to the availability of resources):

1. Continuum Mechanics II
2. Number Theory and Equations over Finite Fields
3. Fuzzy sets & Their applications

**List of Advanced Papers

Each year, Department will offer some courses from the following list of modules, subject to the availability of resources. Four papers have to be chosen by a candidate from the offered modules, keeping in view the prerequisites and suitability of the combination.

Advanced Paper I:

1. Advanced Functional Analysis
2. Advanced Real Analysis
3. Advanced Complex Analysis
4. Advanced Topology I

Advanced Paper II:

1. Mathematical Biology
2. Solid Mechanics
3. Advanced Operations Research
4. Algebraic Topology

Advanced Paper III:

1. Plasma Dynamics
2. Advanced Fluid Dynamics
3. Theory of Waves in Solids
4. Commutative Algebra

Advanced Paper IV:

1. Quantum Mechanics
2. Advanced Dynamical Systems and Chaotic Dynamics
3. Harmonic Analysis
4. Advanced Topology II

Programme Objectives (POs):

The M.Sc. Mathematics programme's main objectives are

- To inculcate and develop mathematical aptitude and the ability to think abstractly in the student.
- To develop computational abilities and programming skills.
- To develop in the student the ability to read, follow and appreciate mathematical text.
- Train students to communicate mathematical ideas in a lucid and effective manner.
- To train students to apply their theoretical knowledge to solve problems.
- To encourage the use of relevant software such as MATLAB and MATHEMATICA.

Programme Specific Outcomes (PSOs):

Successful completion of the two-year M.Sc. course in Mathematics will enable the students to

1. Form a strong foundation in core areas of Mathematics, both pure and applied.
2. Approach and analyse the problems arising in their chosen careers in a logical manner and apply these skills to any real-life situation.
3. Apply computational and modelling skills to specific tasks, especially in the emerging and developing processes and industries.
4. Independently pursue research work in any area of Pure or Applied Mathematics; work in a group confidently and contribute significantly to any research project.
5. Acquire a systematic knowledge of fundamental aspects of various branches of Mathematics which would help them in qualifying National and State-level examinations
6. Think and analyse independently, and apply their skills in mathematical logic to any profession of their choice.
7. Take up pedagogy in Mathematics or related subjects if they are so inclined.

Semester : I

<u>Course : MTM2P COR 01T</u>

Algebra : 50 Marks (4 CP)

Syllabus :

Unit 1 : Normal and subnormal series, Composition series, Jordan-Holder theorem, solvable groups, Nilpotent groups.

Unit 2 : Extension fields, the degree of an extension field, finite extension field, algebraic element. Splitting field, examples of splitting fields, algebraic and transcendental extensions and their characterizations, fundamental theorem of field theory, algebraic closure and algebraically closed field, fundamental theorem of algebra. Some impossibility theorem in the ruler and compass constructions proved as an application to algebraic extensions.

Unit 4 : Group of automorphisms , fixed field, group of automorphisms G(K; F) of a field K relative to its subfield F; Galois group of a polynomial f(x) over a field F and its exercises; separable and normal extension of a field, fundamental theorem of Galois theory , theorem concerning the solvability by radicals of roots of a polynomial and solvability of its Galois group. Insolvability of a Quintic.

Unit 5 : Prime Field and its representation, Finite fields or Galois fields, classification of finite fields, Structure of finite fields, properties of finite fields, isomorphism between two finite fields having the same number of elements.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :
i) Jordan Holder Theorem, Solvable groups, nilpotent groups
ii) Basics of Field extension, algebraically closed field, fundamental theorem of algebra
iii) Group of automorphisms , fixed field, solvability by radicals, Insolvability of a Quintic
iv) Basics of Galois theory , determination of Galois groups.
v) Finite fields or Galois fields, classification of finite fields, Structure of finite fields,

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Algebra, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References:

1. I.N. Herstein, Topics in Algebra, Wiley Eastern Limited, India, 1975
2. Joseph A. Gallian, Contemporary Abstract Algebra, 4th Ed., 1999.
3. Dummit and Foote, Abstract Algebra, 3rd ed. Wiley, New York, 2003.

4. John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.
5. M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.
6. P.B.Bhattacharya, S. K. Jain and S.R. Nagpaul, Basic Abstract Algebra, Cambridge University Press

Course : MTM2P COR 02T

Linear Algebra : 50 Marks (4 CP)

Syllabus :

Modules, Submodules, Quotient Modules, Isomorphisim Theorems, Correspondence Theorem, Exact Sequence, four lemma and five lemma, Simple Modules, Free modules, Modules with chain conditions (Noetherian and Artinian), Dual Modules, Fundamental Structure Theorem for Fintely Generated Modules over PID- Statement only.

Minimal Polynomial, Characteristic Polynomials, Diagonalization of Matrices, Reduction to Triangular Forms, Jordan Blocks, Jordan Canonical Forms, Determinant divisors and Invariant Factors, Rational Canonical Forms, Smith Normal Form Over an Euclidean Domain.

Bilinear Forms, Quadratic Forms, Hermitian Forms, Positve Definite Hermitian Forms & its Direct sum decomposition theorem, Pricipal Minor Criteron, Signature, Sylvester Law of Inertia, Simultaneous Reduction of Pair of Forms.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge on the following :

i) Modules, Quotient Modules, Isomorphisim Theorems, Exact Sequence

ii) Free modules, Modules with chain conditions (Noetherian and Artinian), Dual Modules

iii) Minimal Polynomial, Diagonalization of Matrices, Reduction to Triangular Forms,

iv) Jordan Canonical Forms, Rational Canonical Forms, Smith Normal Form,

v) Bilinear Forms , Quadratic Forms, Hermitian Forms,

vi) Direct sum decomposition theorem, Principal Minor Criterion,

vii) Sylvester Law Of Inertia, Simultaneous Reduction of Pair of Forms.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Linear Algebra, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References:

1. T.S. Blyth, Module Theory an approach to linear algebra- 2 nd edition, Oxford Science Publication, Oxford University Press.

2. K. Hoffman and R. Kunze, Linear Algebra, Pearson Education (India), 2003. Prentice- Hall of India, 1991.

3. S. Lang, Linear Algebra, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1989.

4. A.R. Rao, P. Bhimashankaram, Linear Algebra. (Tata Mc-Graw Hill)

5. P. Lax, Linear Algebra, John Wiley & Sons, New York, Indian Ed. 1997

6. H.E. Rose, Linear Algebra, Birkhauser, 2002.

7. S. Lang, Algebra, 3rd Ed., Springer (India), 2004.

8. G. Strang : Linear Algebra & its Applications, Harcourt Brace Jovanichn 3rd Edition 1998.

9. B. Noble and J.W. Daniel. Applied Linear Algebra, third edition, 1988.Prentice Hall, NJ.

10. N.J. Pullman. Matrix Theory and its Applications, 1976. Marcel Dekker Inc. New York.

11. I. N. Herstein, Topics in Algebra.

12. R. Stall, Linear Algebra and Matrix Theory.

13. Evar D. Nering, Linear Algebra and Matrix Theory.

14. B. C. Chatterjee, Linear Algebra.

15. S. Kumaresan, Linear Algebra- A Geometric Approach, Prentice Hall of India, 1999.

16. I. S. Luthar and I. B. S. Passi, Field Theory (Algebra, Vol. 4), Narosa Publishing House, Kolkata, 2010.

17. Seymour Lipschutz, 3,000 Solved Problems in Linear Algebra

18. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra, 4th Ed., Prentice Hall of India Pvt. Ltd., New Delhi, 2004.

Course : MTM2P COR 03T

Real Analysis: 50 Marks (4 CP)

Syllabus :

Functions of bounded variation**:** Brief discussions of its basic properties, Nature of points of discontinuity, Nature of points of non-differentiability, positive and negative variation and their properties.

The Lebesgue measure**:** Lebesgue Outer measure, countability, subadditivity, measurable sets and their properties, non-measurable sets, definition of Lebesgue measure and its basic properties.

Measurable functions**:** Definition on a measurable set in R and basic properties, Simple Functions.

Absolutely continuous functions**:** Definition and basic properties, Deduction of the class of all absolutely continuous functions as a proper subclass of all functions of bounded variation; Characterization of an absolutely continuous function in terms of its derivative vanishing almost everywhere, property (N), every absolutely continuous function possesses the property (N).

Differentiation on R^n**:** Functions from $R^n$to R^m, projection functions, component functions, scalar and vector fields, open balls and open sets, limit and continuity. Derivative of a scalar field with respect to a vector, directional derivatives and partial derivatives, partial derivatives of higher order, Chain rule, Frechet derivative, matrix representation of derivative of functions, continuously differentiable functions, Implicit function theorem, inverse function theorem.

Integration on R^n**:** Integral of f**:**$A \rightarrow R$ when $A \subset R^n$ is a closed rectangle. Conditions of integrability. Integrals of f**:** $C \rightarrow R$, $C \subset R^n$ is not a rectangle, concept of Jordan measurability of a set in R^n. Fubini's theorem for integral of f**:**$A \times B \rightarrow R$, $A \subset R^n$, $B \subset R^n$, are closed rectangles. Fubini's theorem for f**:** $C \rightarrow R^n$, $C \subset A \times B$. Formula for change of variables in an integral in R^n.

Course Outcomes: Upon completion of this course, the student will be able to understand the basics of Real Analysis and improve the logical thinking.

References:

1. T. M. Apostol, *Mathematical Analysis,* NarosaPubli. House, 1985.
2. H. L. Royden, *Real Analysis*- 3rd Edn, Pearson, 1988
3. J. C. Burkil& H. Burkil, *A second Course of Mathematical Analysis,* CUP, 1980.
4. R. R. Goldberg, *Real Analysis,* Springer-Verlag, 1964.
5. I.P.Natanson, *Theory of Functions of a Real Variable,* Vol. I, Fedrick Unger Publi. Co., 1961.
6. W. Rudin, *Principle of Mathematical Analysis,* Mc Graw Hill, N.Y., 1964.
7. Charles Swartz, *Measure, Integration and Function Spaces*, World Scientific, 1994.
8. M. Spivak, *Calculus on Manifolds,* The Benjamin/Cummings Pub. comp., 1965.
9. J. R. Munkres, *Analysis on manifolds,* Addison-Wesley Pub. Comp., 1991.
10. R. Courant and F. John, *Introduction to Calculus and Analysis, Vol – II,* Springer Verlag, New York, 2004.

Course : MTM2P COR 04T

Gr. A: Graph Theory Gr. B: Special and Generalised Functions

Syllabus :

Gr. A: Graph Theory : 25 Marks (2 CP)

Undirected graphs, Directed graphs, Geometrical representation of graphs, Handshaking lemma due to Euler and some basic properties of a graph. In - degree and out - degree of a vertex in a digraph. Simple digraph and underlying graph. Representation of binary relations on finite sets by digraphs. Reflexive, symmetric and transitive digraphs.

Sub graph, spanning sub graph, induced sub graph on a vertex set and induced sub graph on an edge set. Isomorphism of graphs. Walks, paths, circuits and cycles with their properties, concatenation of two walks.

Connected and disconnected graphs. Component of a graph, decomposition of a graph into finite number of components, acyclic graph and cycle edge of a graph. properties of connected graphs. Complete graphs, disconnecting sets, bridge, separating sets, distance between two vertices of a graph. Complement of a graph, Self complementary graphs, Ramsey problem. Bipartite graphs, complete bipartite graphs, necessary and sufficient condition for a simple graph to be bipartite.

Eulerian and Hamiltonian graphs: Euler trials, Euler circuits, Edge traceable graphs, Euler graphs, Euler's Theorem. Fleury's algorithm, Konigsberg bridge problem. Hamiltonian path, Hamiltonian cycle, Hamiltonian graph. Dirac's Theorem, Ore's Theorem (statements only) and its use.

Trees and forests with their properties. Minimally connected graphs, spanning trees. weighted graphs, Kruskal's algorithm for a minimal spanning tree. Rooted tree, binary tree.

Matrix representation of graphs, adjacency matrices of graphs and digraphs and their properties, path matrix, incidence matrices of graphs and digraphs and their properties.

Course Outcomes : After the course the student will have a strong background of graph theory. The students will be able to apply principles and concepts of graph theory in practical situations such as computer science, physical and engineering sciences.

References :

1. N. Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India (2000).
2. J. P. Tremblay & R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, Mcgrow-Hill Book Co. (1997).
3. F. Harary, Graph Theory, Addison-Wesley Publishing Company (1972).
4. J. Gross & J. Yellen, Graph Theory and its Applications, CRC Press (USA) (1999).

Gr. B: Special and Generalised Functions :25 Marks (2 CP)

Concepts of ordinary and singular points of a second order linear differential equation in a complex plane, Fuch's theorem, Solution at an ordinary point, Regular singular point, Frobenius' Method, Solution at a regular singular point, Series solutions of Legendre and Bessel equations.

Legendre polynomial: Generating function, Schlaffli's integral,Rodrigue's formula, recurrence relations, orthogonality property, expansion of a function in a series of Legendre polynomials.

Bessel function of 1^{st} kind and its properties.

Definition of test functions. Generalised Functions: Regular and Singular. Delta function and Delta convergence. Product and Derivative of generalized functions. Slow growth generalized functions. Fourier Transforms. Fourier series of periodic generalized functions. Powers of $|x|$ as generalised functions, Even and odd generalized functions, Multiplication and integration of generalized functions.

Course Outcomes: The students will acquire knowledge about generalized and special functions and their importance. They will be able to solve problems of several types arising in applied mathematics and theoretical physics using generalized and special functions.

References :

1. Generalized Functions, Volume 1: Properties and Operations. I. M. Gel'fand& G. E. Shilov (American Mathematical Society, 2016).

2. R.F. Hoskins, Generalized functions, Horwood, Chichester and New York, 1979.

3. R.P. Kanwal, Generalized Functions: Theory and Technique, Birkhauser, New York, 1998.

4. D.S. Jones, Generalized Functions, Cambridge University Press, 1982.

5. An Introduction to Fourier Analysis and Generalized Functions, M.J. Lighthill.

6. V. S. Vladimirov, Methods of the Theory of Generalized Functions. (CRC Press, 2002).

7. I.N. Sneddon, Special functions of Mathematical Physics and Chemistry, Longman, 1980.

8. G. F. Simmons, Differential Equations, TMH, 2006.

9. E.A. Coddington and N. Levison, Theory of Ordinary Differential Equations, Tata McGraw Hill, 1955.

10. E.L. Ince, Ordinary Differential Equations, Dover, 1956.

11. E.D. Rainville, Special Functions, MacMillan, 1960.

12. N.N. Labedev, Special Functions, and their applications, Dover, 1972.

Course : MTM2P COR 05T

Classical Mechanics : 50 Marks (4 CP)

Syllabus :

Introduction, Kinematics for a single particle.Laws of Motion for a particle.

Conservation principles. Principle of energy. Central forces and central orbits.

System of particles. Generalised Co-ordinates. Constraints, unilateral, bilateral, holonomic, schleronomic, rheonomic. Principle of Virtual Work. D'Alembert's Principle. Lagrange's equations for Holonomic and Nonholonomic Systems. Lagrange's Equation of Motion. Energy Equation for Conservative Fields. Cyclic or Ignorable Co-ordinates. Routh's Equations.

Hamilton's Equations of Motion. Calculus of Variations. Hamilton's Principle. Hamilton's and Lagrange's Equations of Motion form Hamilton's Principle. Principle of Least Action. Constants of Motion. Noether's Theorem. Conservation Laws. Dynamical systems. Liouville's theorem for conservative flow.

Motion of a Rigid Body. Euler's Theorem. Motion about a Fixed Point in it. Euler's Dynamical Equations. Motion of a symmetric top in absence of torque. Eulerian angles. Motion of a Symmetrical top under gravity. Stability of Steady Precession.

Canonical Transformations. Generating Functions. Poisson's Bracket. Jacobi's Identity. Poisson's Theorem. Jacobi-Poisson Theorem.

Hamilton-Jacobi Partial Differential Equation. Jacobi's Theorem. Hamilton's Principal Function. Hamilton's Characteristic Function. Action Angle Variables. Adiabatic Invariance.

Theory of Small Oscillations (Conservative System). Normal Co-ordinates. Oscillations under Constraints. Stationary Character of Normal Modes.

Course Outcomes :

On completion of the course ,

1. Students will understand the basic principles of mechanics and have a clear idea of conservative and non-conservative force-fields, constraints , conservation of energy.
2. Students will be able to apply the equations of motion to solve analytically the problems of motion of a single particle/a system of particle or rigid body under conservative force fields.
3. Will learn about a new approach through the Hamilton's principle for deriving the equations of motion of a system.
4. Gain knowledge of Hamiltonian system and phase planes from the point of view of mechanics.
5. Use the theory of normal modes for solving problems related to oscillations and vibrations.
6. Students will learn the basics of classical mechanics required for further studies in solid and quantum mechanics.

References :

1. 1. H. Goldstein, *Classical Mechanics*. Narosa Publishing House, New Delhi, (1980).

2. F. Gantmacher, *Lectures in Analytical Mechanics,* MIR Publishers, Moscow, (1975).

3. J. L. Synge and B.A. Griffith, *Principles of Mechanics*, McGraw-Hill, N.Y. (1970).

4. N. C.Rana and P. S. Joag, *Classical Mechanics*, Tata McGraw Hill Pub. Company Ltd., New Delhi, (1998).

5. N. H. Louis and Janet D. Finch, *Analytical Mechanics*, C.U.P., (1998).

6. E .T. Whittaker, *A Treatise of Analytical Dynamics of Particle and Rigid Bodies,* C.U.P., (1977)

7. A. S. Ramsey, *Dynamics Part-II,* C.U.P.

8. V. I. Arnold, *Mathematical Methods of Classical Mechanics*, 2nd ed., Springer-Verlag, (1997).

9. N. G. Chetaev, *Theoretical Mechanics*, Springer-Verlag, (1990).

10. F Chorlton, *Text Book of Dynamics*, CBS Publishers, (1985).

11. L. D. Landau and E.M. Lifshitz, *Mechanics*, 3rd ed., Pergamon Press, (1982).

12. J R Taylor, *Classical Mechanics.*
13. K A I L W Gamalath , *Introduction to Analytical Mechanics* ,Narosa, 2011.
14. L N Katkar, *Problems in Classical Mechanics* ,Narosa 2013.
15. Madhumangal Pal, *A Course on Classical Mechanics* , Narosa, 2009.
16. S Deo, R Rahman, *Classical Mechanics : An Introduction*, Narosa, 2023.
17. N M J Woodhouse, Introduction to Analytical Dynamics ,Springer(London),2009.
18. Grant R Fowles, George L Cassiday, *Analytical Mechanics*, Thomson Brookes/Cole USA, 2005.
19. Louis N Hand, Janet D Finch , *Analytical Mechanics*, CUP,1998.
20. Classical Mechanics, R Douglas Gregory, *Classical Mechanics*, CUP, 2001.

Course : MTM2P AEC 01M

Computer Aided Numerical Analysis with Python
50 Marks (2 CP)

Syllabus :

Python programs on Numerical methods for finding solution of algebraic and transcendental equations, Interpolation, Numerical differentiation & Numerical integration and Numerical solutions of ordinary and partial differential equations.

Example of programming problems: Bisection Method, Newton Raphson Method, Regula Falsi Method, Trapezoidal Rule for integration, Simpsons 1/3rd rule, Euler's method for ODE, RK method of ODE etc.

Course Outcomes: At the end of this course a student should be able to :

- understand the purpose of basic computer programming language,
- understand and apply control statements, implementation of arrays, functions, etc.,
- enhance ability to program writing skills for solving several real life and Mathematical problems,

References :

1. W.J. Chun, Core Python Programming, Prentice Hall (2013).
2. Kenneth A. Lambert, Fundamentals of Python, Cengage (2015).
3. E. Balagurusamy, Introduction to Computing and Problem Solving Using Python, McGraw Hill India (2016).
4. Timothy A. Budd, Exploring Python, McGraw Hill India.
5. Jaan Kiusalaas, Numerical Methods in Engineering with Python 3, Cambridge University Press.
6. Qingkai Kong, Timmy Siauw, Alexandre Bayen Python Programming And Numerical Methods: A Guide For Engineers And Scientists

Semester : II

<u>Course : MTM2P COR 06T</u>

Topology : 50 Marks (4 CP)

Syllabus :

Topological spaces, Open and Closed sets, Bases and sub-bases. Closure and Interior – their properties and relations; Exterior, Boundary, Accumulation points, Derived sets, Adherent point, Dense set, G_δ and F_σ sets. Neighboourhoods and neighbourhood system.

Alternative methods of defining a topology in terms of Kuratowski closure operator, interior operator, neighbourhood systems.

Subspace and Induced or Relative topology. Relation of closure, interior, accumu-lation points etc. between the whole space and the subspace.

Continuous, open and closed maps, pasting lemma, homeomorphism and topological properties.

1 st and 2nd countability axioms, Separability, Lindeloffness and their relationships. Characterizations of accumulation points, closed sets, open sets in a 1st countable space w.r.t. sequences , Heinei's continuity criterion .

Ti spaces(i = 0, 1, 2, 3, 3½, 4, 5), their characterizations, basic properties and mutual relations. Urysohn's lemma and Tieze' extension theorem (statement only) and their applications.

Connected and disconnected spaces. Connectedness on the real line. Components and quasi-components.

Compactness, its basic properties and characterizations, Alexander subbase theorem, Continuous functions and compact sets, Compactness and separation axioms . Equivalence of compactness, countable compactness and sequential compactness in metric spaces.

Product and box topology, Projection maps. Tychonoff product theorem. Separation and product spaces. Connectedness and product spaces. Countability and product spaces.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify,

demonstrate and explain the acquired knowledge on the following :

i) Basics of Topological spaces, Relative topology, continuity, homeomorphism and topological properties ,

ii) Alternative methods of defining a topology in terms of Kuratowski closure operator, interior operator and neighbourhood systems,

iii) Countability axioms, Heinei's continuity criterion,

iv) Lower & higher separation axioms, Urysohn's lemma and Tieze's extension theorem (statement only) and their applications,

v) Connected and disconnected spaces, Components and quasi-components.

vi) Compactness, Alexander subbase theorem, equivalence of various compactness in metric spaces,

vii) Product and box topology, Tychonoff product theorem,

Also there is a scope, for applying the acquired knowledge of the above topological methods/ tools, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. General Topology by J. L. Kelley, Van Nostrand
2. General Topology by S. Willard, Addison-Wesley.
3. Topology by J. Dugundji, Allyn and Bacon.
4. Topology, A first course by J. Munkres, Prentice Hall, India.
5. Introduction to topology and modern analysis by G. F. Simmons, McGraw Hill.
6. Introduction to General topology by K. D. Joshi, Wiley Eastern Ltd.
7. General Topology by Engelking, Polish Scientific Publishers, Warszawa.
8. Counter examples in Topology by L. Steen and J. Seebach.
9. A text book of Topology by B. C. Chatterjee, S. Ganguly and M. Adhikari, Asian Books Pvt.

Course : MTM2P COR 07T

Functional Analysis: 50 Marks (4 CP)

Syllabus :

Review of Metric spaces: Metric Spaces, Hölder and Minkowski inequalities (statement only), continuity and uniform continuity, completeness, compactness, connectedness.Baire's category theorem, Banach's fixed point theorem and its applications to solutions of certain systems of linear algebraic equations, Picard's existence theorem on differential equation, Implicit function theorem and Fredholm's integral equation of the second kind, Kannan's fixed point theorem.

Real and Complex linear spaces. Normed induced metric. Banach spaces, the spaces R^n, C^n, C [a, b], C_0, C, lp(n)(1≤p≤∞), lp(1≤p≤∞) and $L_2[a,b]$. Riesz's lemma. Finite dimensional normed linear spaces and subspaces, completeness, compactness criterion, Quotient space, equivalent norms and its properties.

Bounded linear operators, various expressions for its norm. Spaces of bounded linear operators and its completeness. Inverse of an operator. Linear and sublinear functionals, Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces and some of its simple applications.

Conjugate or Dual spaces, Examples, Separability of the Dual space. Reflexive spaces, weak and weak* convergence. Uniform boundedness principle and its applications. The Open mapping Theorem and the Closed graph Theorem.

Inner product spaces and Hilbert spaces, examples of Hilbert spaces, continuity of inner product, C-S inequality, basic results on Inner product spaces and Hilbert spaces, parallelogram law, Pythagorean law, Polarization identity, orthogonality, orthonormality, orthogonal complement. The Riesz representation theorem, Bessel's inequality. Convergence of series corresponding to orthogonal sequence, Fourier coefficient, Perseval identity. Riesz- Fischer Theorem.

Course Outcomes: On successful completion of this course, students will be able to appreciate how functional analysis uses and unifies ideas from vector spaces, the theory of metrics, and complex analysis. Moreover, students will be able to understand and apply fundamental theorems from the theory of normed and Banach spaces, Hilbert spaces.

References:

1. W.Rudin, *Functional Analysis*, Tata McGraw Hill.
2. B.V.Limaye, *Functional Analysis*, Second Edition, New Age – International limited, Madras.
3. G. Bachman & L. Narici,*Functional Analysis*, Academic Press, 1966.
4. N. Dunford & J. T. Scwarz, Linear operators, Vol – I & II, Interscience, New York, 1958.
5. L. V. Kantorvich and G.P. Akilov, *Functional Analysis*, Pergamon Press, 1982.
6. E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley Eastern, 1989.
7. I. J. Madox, *Elements of Functional Analysis*, Universal Book Stall, 1992.
8. A. H. Siddiqui, K. Ahmed and P. Manchanda, *Introduction to Functional Analysis with applications*, Anshan Publishers, 2007.
9. A. E. Taylor, *Functional Analysis*, John wiley and Sons, New York, 1958.

Course : MTM2P COR 08T

Electromagnetic Theory and Special Theory of Relativity: 50 Marks (4 CP)

Syllabus :

Electrostatics: Coulomb's law, Electric Field, Divergence and Curl of Electrostatic Fields, Gauss' law, Electric Potential: Poisson and Laplace equation, Work and Energy, Conductors.

Electric Fields in Matter: Polarization, Electric Displacement, Linear Dielectrics.

Magnetostatics: Lorentz Force Law, Steady currents, Biot-Savart Law, Divergence and Curl of B, C Magnetic Vector Potential. Magnetic Fields in Matter: Field of a Magnetized Object, Ampere's Law in Magnetized Material, Linear and Nonlinear Media.

Electromagnetic Induction: Faraday's Law, Maxwell's Equations. Conservation Laws, Continuity Equation, Poynting's Theorem. Newton's Third Law in Electrodynamics, Maxwell's Stress Tensor, Conservation of Momentum.

Electromagnetic Waves in Vacuum and Matter, Fresnel's equations, Absorption and Dispersion, Guided Waves. Coulomb Gauge and Lorenz Gauge, Jefimenko's Equations, Dipole radiation.

Galelian transformations ,Postulates of special relativity, Invariance of space-time line element, Lorentz transformations, length contraction, simultaneity, time dilation.

Lorentz invariants. Velocity transformation laws. Relativistic Doppler's effect. Relativistic mass and energy. Force and acceleration in relativity. 4- vectors, 4-velocity,4-acceleration and 4-momentum.

Relativistic mass. Momentum and energy conservation in STR, Collision, 4-force. Light cone, time like-light and space-like vectors. Relativistic Lagrangian and Hamiltonian.

Course Outcomes:

The students will acquire knowledge about Electromagnetic Theory and Special Theory of Relativity along with their usefulness for practical purpose. They will learn how different laws of electricity and magnetism can be put together to derive the Maxwell' s equations. Further they will build up strong application capability of graduate level mathematics, understand and apply the basic theories of electromagnetism and grow interest in electrical engineering. They will understand the difference between classical and relativistic mechanics also.

References :

1. D.J. Griffiths, Introduction to Electrodynamics, Prentice Hall, New Delhi.

2. L. D. Landan and E. M. Lifshitz, The classical Theory of Fields.

3. A. Sommerfield, Electrodynamics.

4. J.D. Jackson, Classical Electrodynamics.

5. J. H. Jeans, Mathematical Theory of Electricity and Magnetism, Cambridge University Press.

6. V. S. A. Ferraro, Electromagnetic Theory, Athlone Press, London.

7. I. E. Irodov, Basic laws of Electromagnetism, CBS.

8. A. Einstein, Relativity: The Special and the General Theory

9. R. Resnick: Introduction to special relativity.

10. F. Rahaman: The Special Theory of Relativity: A Mathematical approach.

Course : MTM2P COR 09T

Integral Equations and Calculus of Variations : 50 Marks (4 CP)

Syllabus :

Integral Equations

Definition of Integral Equation and their classification. Reduction of differential equation to integral equation and vice-versa. Eigen values and Eigen functions.

Existence and uniqueness of solutions of Fredholm and Volterra integral equations of second kind. Solutions by the method of successive approximations, series solution. Iterated kernels. Resolvent kernels. Solution of integral equations with separable kernels. Solution of Volterra integral equation of first kind. Neumann series.

Fredholm theorems and Fredholm Alternatives. Hilbert-Schmidt theorem. Non-homogeneous Fredholm integral equation with symmetric kernel. Applications of Green's function for solution of the boundary value problems. Integral equation formulations of boundary value problems with more general and inhomogeneous boundary conditions. Integral equation formulation of boundary value problems for Laplace's equation. Poisson's integral formula.

Singular Integral equation, Solution of Abel's Integral equation. Solution of Volterra equation of convolution type by Laplace transform. Solutions of Integro-differential equation.

Calculus of Variations

Functional and its variation, Necessary and sufficient conditions for extremum, Euler's equation, The Brachistochrone problem, geodesic. Extremals, Variational problems with functional dependent on functions of several independent variables. Lagrange's problem.

Variational problems: Parametric and Isoperimetric problems and Applications.

Course Outcomes: After completing this course, the student will be able to:

- distinguish between differential and integral equations,
- understand the theory of existence and uniqueness of solutions of linear integral equations,
- find solutions of linear integral equations of first and second type (Volterra and Fredhlom) and singular integral equations using several techniques.
- understand the concept of variational problems and their applications.

References :

1. R. P. Kanwal - Linear Integral Equation – Theory and Techniques, Academic Press, New York (2012).
2. W.V. Lovitt - Linear Integral Equations, Dover, New York.
3. A. Wazwaz - A first course in integral equations, World Scientific, (1997).
4. F. G. Tricomi - Integral Equations, Dover.
5. S. G. Mikhlin - Integral Equations, Pergamon Press (1960).
6. A.S. Gupta - Calculus of Variations with Applications, Prentice Hall (1997).
7. I. M. Gelfand and S. V. Fomin, Calculus of Variations, Dover Publications (2000).

<u>Course : MTM2P DSE 01T</u>

(Elective Paper I*)

1. Continuum Mechanics I: 50 Marks (4 CP)

Syllabus :

Introduction. Idea of continuum: Continuum hypothesis, Continuous media, Body, Configuration, continuum motion as a real continuous map. Material and spatial time derivatives.

Theory of deformation and strain: Deformation and flow, Lagrangian and Eulerian descriptions, Deformation gradient tensors, Finite strain tensor, Finite strain components in rectangular Cartesian coordinates, Small deformation, Infinitesimal strain tensor, Infinitesimal strain components, Geometrical interpretation of infinitesimal strain components, Strain quadric of Cauchy, Principal strains, Strain invariants, Compatibility equations for linear strains.

Rate of strain tensors-its principal values and invariants, Rate of rotation tensor, vorticity vector, velocity gradient tensor.

Theory of stress: Forces in a continuum, Stress tensor, Equations of equilibrium, Symmetry of stress tensor, Shearing and normal stresses, Stress quadric of Cauchy and its properties. Maximum shearing stress, Principal stresses and principal axes of stresses, Invariants of stress tensors, Stress compatibility equations.

Motion of a continuum: Principle of conservation of mass, The continuity equation, Principles of conservation of linear and angular momentum, and principle of energy.

Theory of elasticity: Ideal materials, Classical elasticity, Generalized Hooke's Law, Isotropic and anisotropic materials, Constitutive equation for isotropic elastic solid, and anisotropic solids. Elastic modulii, Strain-energy function, Physical interpretation. Equation of motion for linear isotropic solids.

Constitutive equations for Newtonian Fluid. Stress and rate of strain relation. Navier-Stokes' Equation.

Equations of equilibrium and motion in terms of displacement. Fundamental boundary value problems of elasticity and uniqueness of their solutions (statement only). Saint-Venant's principle – solution of simple problems.
Wave propagation in an infinite elastic medium, Waves of distortion and dilatation.

Course Outcomes:

On completion of this course ,

- *The students would be able to apply ideas of continuum mechanics to analyse both solid and fluid motion.*
- *This course will prepare the students for further courses on fluid and/or solid dynamics, geophysics and plasma physics.*

References :

1. A. C. Eringen - Mechanics of Continua, Wiley, 1967.
2. C.S. Jog , Continuum Mechanics, Volume I : Foundations and Applications of Mechanics(Cambridge_IISC) .
3. S. K.Bose - Continuum Mechanics Theory, Narosa, 2017.
4. S.W. Yuan - Foundations of Fluid Mechanics, Prentice – Hall International, 1970.
5. D. S. Chandrasekharaiah and L. Debnath- Continuum Mechanics, Academic Press, 1994.
6. Leigh, D. C. – *Non-Linear Continuum Mechanics* (MacGraw-Hill)
7. Truesdell, C – *Continuum Mechanics*
8. Chung, T. J. – *Contunuum Mechanics* (Prentice-Hall)
9. Truesdell, C and Nol, W. – *Encyclopaedia of Physics.* Vol. III/3, 1965 (Ed. S. Flugge)
10. . Milne – Thomson, L. M.- *Theoretical Hydrodynamics*
11. Eringen, A. C. – *Non-linear Theory of Continuous Media* (MacGraw-Hill)
12. I. S. Sokolnikoff - Mathematical theory of Elasticity, Tata Mc Grow Hill Co., 1977.

2. Differential Manifold : 50 Marks (4 CP)

Syllabus :

Differentiable manifolds: basic notions; the effects of second countability and Hausdorffness; tangent and cotangent spaces; submanifolds; consequences of the Inverse Function Theorem; vector fields and their flows; the Frobenius Theorem; Sard's theorem.

Differential forms: Multilinear algebra; tensors; differential forms; the de Rham complex and its behaviour under differentiable maps; the Lie derivative; differential ideals.

Lie groups: Lie groups; Lie algebras; homomorphisms; Lie subgroups; coverings of Lie groups; the exponential map; closed subgroups; the adjoint representation; homogeneous manifolds.

Integration on manifolds: orientation; the integral of differential forms on differentiable singular chains; integration of differential forms of top degree on an oriented 3 differentiable manifold; the theorems of Stokes; the volume form on an oriented Riemannian manifold; the divergence theorem; integration on a Lie group.

de Rham cohomology: definition; real differentiable singular cohomology; statement of the de Rham theorem; the Poincar lemma.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following:

- i) tangent and cotangent spaces; submanifolds,
- ii) vector fields and their flows; the Frobenius Theorem,
- iii) multilinear algebra, differential forms, the Lie derivative,
- iv) Lie groups and Lie algebras,
- v) Integration on manifolds, theorems of Stokes, integration on a Lie group,
- vi) de Rham cohomology.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Differentiable manifolds to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions.

References :

1. M. Spivak, A Comprehensive Introduction to Differential Geometry, Vols I-V; Publish or Perish, Inc. Boston,1979.
2. J.A. Thorpe, Elementary topics in Differential Geometry, Under - graduate Texts in Mathematics, Springer – Verlag, 1979.
3. S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, Interscience Publishers, 1963.
4. F. W. Warner, Foundations of differentiable manifolds and Lie groups.
5. Christian Br, Elementary Differential Geometry; Cambridge University Press, 2011.
6. I. Madsen and J. Tornehave, From calculus to cohomology, Cambridge University Press.

3. Operations Research: 50 Marks (4 CP)

Syllabus :

Revised simplex method, Dual simplex method, Post optimal analysis.

Nonlinear programming : Karush-Kuhn-Tucker necessary and sufficient conditions of optimality, Convex programming. Quadratic programming, Wolfe's method, Beale's method.

Dynamic programming : Bellman's principle of optimality, Recursive relations, System with more than one constraint, Solution of LPP using dynamic Programming.

Inter programming : Gomory's cutting plane method , Branch and bound method.

Sequencing Models : The mathematical aspects of Job sequencing and processing problems, Processing n jobs through Two machines, processing n jobs through m machines.

Inventory control : Concept of EOQ, Problem of EOQ with finite rate of replenishment, Problem of EOQ with shortages, Multi-item deterministic problem, Probabilistic invontory models.

Queueing Theory : Basic features of queueing systems, operating characteristics of a queueing system, arrival and departure (birth & death) distributions, inter-arrival and service times distributions, transient, steady state conditions in queueing process. Poisson queueing models-M/M/1, M/M/C for finite and infinite queue length.

Course Outcomes: After completing this course, the student will be able to :

- solve nonlinear programming problems using Lagrange multiplier, Kuhn-Tucker conditions, Wolfe's and Beale's method,
- find optimal solution of dynamic programming problem,
- learn theory of sequencing models and inventory control and their applications,
- understand Queueing Theory and its applications,
- identify and formulate some real life problems into nonlinear programming problem.

References :

1. H. A. Taha - Operations Research-An Introduction. Macmillan Pub. Co., Inc., New York.
2. G. Hadley -Nonlinear and Dynamic Programming, Addition-Wesley.
3. S. S. Rao - Optimization Theory and Application, Wiley Eastern.
4. K Sarup, P. K. Gupta and Man Mohan - Operation Research, Sultan Chand & Sons.
5. J. K. Sharma - Operation Research, Mcmillan, India.
6. F. S. Hillier and G. J. Lieberman- Introduction to Operations Research, TMH, 2008
7. S. D. Sharma-Operation Research, Kedarnath & Ramnath, Meerat.
8. O. L. Mangasarian-Non linear Programming, McGraw Hill.
9. R. Panneerselvam - Operations Research, PHI, 2009.

Semester : III

Course : MTM2P COR 10T

Measure and Integration : 50 Marks (4 CP)

Syllabus :

Outer Lebesgue Measure m* in the Euclidean line and its Properties. Outer measure **μ *** on S, where S is a space; the concept of μ-measurable sets with the help of **μ *** . Necessary and sufficient condition for μ-measurability. Properties of **μ**-measurable sets. The structure of μ-measurable sets-the concept of σ-algebra; the σ-algebra of Lebesgue measurable sets.

Properties of Lebesgue measure, Vitali's theorem: The existence of an non-measurable set in the Euclidean line . The Borel sets & Lebesgue measurable sets- a comparison .

μ-measurable functions, their properties; Characteristic functions, Simple functions. Theorem relating to the non negative μ-measurable function as a limit of a monotonically increasing sequence of non negative simple μ-measurable functions.

Lebesgue Integration : Integration for simple functions and for Extended real valued **μ**-measurable functions; The countable additivity of the set of function νf on **M** defined by νf(M) = ∫M f , for each set M Є **M** , the **σ**- algebra of μ-measurable sets, for a nonnegative **μ**-measurable function f ; Lebesgue's monotone convergence theorem and its applications, Fatou's lemma, Lebesgue's dominated convergence Theorem.

Necessary & Sufficient condition of Riemann integrability via measure; interrelation between the two modes of integration..

The Concept of Lp -spaces; Inequalities of Holder and Minkowski; Completion of Lp -spaces.

Convergence in Measure, Almost Uniform Convergence, Pointwise Convergence a.e and their Characterizations; Convergence Diagrams, Counter Examples. Egoroff theorem.

Lebesgue Integral in the Plane. Product σ-algebra. Product Measure. Fubini's Theorem.

If time permits :

Signed Measure and the Hahn Decomposition; The Jordan Decomposition. The Radon-Nikodym Theorem.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Lebesgue measure, Vitali's theorem concerning existence of non-measurable sets,

ii) measurable functions, Theorem relating to non negative μ-measurable function as a limit of a monotonically increasing sequence of non negative simple μ-measurable functions,

iii) Lebesgue's monotone convergence theorem and its applications, Fatou's lemma, Lebesgue's dominated convergence Theorem,

iv) Interrelation between Riemann & Lebesgue integration,

v) Concept of Lp -spaces and its completeness,

vi) Characterizations of Convergence in Measure, Almost Uniform Convergence, Egoroff theorem,

vii) Product Measure. Fubini’s Theorem,

viii) Signed Measure and the Hahn Decomposition, Radon-Nikodym Theorem.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Measure and Integration, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. P. R. Halmos, Measure Theory, Von Nostrand, New York, 1950.

2. E. Hewitt & K. Stromberg, Real and abstract Analysis, Third edition, Springer-.

3. G. D. Barra, Measure Theory & Integration, Wiley Eastern Limited, 1987.

Verlag, Heidelber & New York, 1975.

4. W. Rudin, Real and Complex Analysis, Tata McGraw- Hill, New York, 1987.

5. I. K. Rana, An introduction to Measure & Integration, Narosa Publishing House, 1997.

6. H. L.Royden, Real Analysis, Macmillan Pub. Co. Inc, New York, 1993.

7. J. F. Randolf, Basic Real and Abstract Analysis, Academic Press, New York, 1968.

8. C. D. Aliprantis and Owen Burkinshaw, Principles of Real Analysis, Academic Press, 2000.

9. K. R. Parthsarathy, introduction to Probaility and Measure, Macmillan Company of India Ltd., Delhi, 1977.

10. R. B. Bartle, Elements of Real Analysis.

Course : MTM2P COR 11T

Operator Theory and Banach Algebra: 50 Marks (4 CP)

Syllabus :

Dual spaces, Representation Theorem for bounded Linear functionals on C[a,b] and L_p spaces, Dual of C[a,b] &L_p spaces, weak & weak*convergence, Reflexive spaces.

Bounded Linear Operators, Uniqueness Theorem, Adjoint of an Operator and its Properties; Normal, Self Adjoint, Unitary, Projection Operators, their Characterizations & Properties. Orthogonal Projections, Characterizations of Orthogonal Projections among all the Projections. Norm of Self Adjoint Operators, Sum & Product of Projections, Invariant Subspaces. Sesquilinear functionals on linear spaces and on Hilbert spaces, generalization of Cauchy-Schwarz inequality.

Spectrum of an Operator, Finite Dimensional Spectral Theorem, Spectrum of Compact Operators, Spectral Theorem for Compact Self Adjoint Operators (statement only).

Algebra and some properties of the space C(X), Stone-Weierstrass Theorem.
Banach Algebra, Banach Sub Algebra, Identity element, invertible elements, existence and representation of the inverse of e-x, resolvent set and resolvent operator, analytic property of the resolvent operators, compactness of spectrum, non-emptiness of the spectrum. Division Algebra, Gelfand-Mazur Theorem. Topological divisors of zero. Spectral radius and its properties, spectral mapping theorem for polynomials. Complex homomorphism, Gleason-Kahane-Zalazko Theorem, Commutative Banach Algebra, Ideals, maximal ideals, Quotient space as a Banach Algebra under certain conditions. Gelfand theory on representation of Banach Algebra, Gelfand transform, weak Topology, weak* Topology, Gelfand Topology, Banach Alaoglu Theorem. Quotient algebra, Banach *-algebra, B* -algebra, Gelfand Naimark Theorem.

Course Outcomes: Students will be able to understand the fundamentals of spectral theory, and appreciate some of its power. Students will have the knowledge and skills to apply problem solving using functional analysis techniques applied to diverse situations in physics, engineering and other mathematical contexts.

References:

1. W.Rudin, *Functional Analysis*, Tata McGraw Hill.
2. Schaffer, Topological Vector Spaces.
3. G. Bachman & L. Narici,*Functional Analysis*, Academic Press, 1966.
4. E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley Eastern, 1989
5. Diestal, Applications of Geometry of Banach Spaces.
6. Horvat, Linear Topological spaces.
7. Brown and Page, Elements of Functional Analysis.

Course : MTM2P COR 12T

Nonlinear Differential Equations and Dynamical Systems : 50 Marks (4 CP)

Syllabus :

System of ODE's Autonomous System, Phase Plane Analysis, Equilibrium Points, Classification of equilibrium points, Stablility of equilibrium points. Nonlinear autonomous systems. Flow diagram, Phase portrait, Isocline. Fixed points and their nature. stability, asymptotic stability, Linearization about a critical point.

Stability. Poincare and Lyapunov stability. Solutions and paths, linear systems, zero solutions of nearly linear systems. Stability through Lyapunov Functions.

Conservative systems. Hamiltonian systems. Index of an equilibrium point. The index at infinity. The phase diagram at infinity. Homoclinic and heteroclinic paths. Limit cycles and other closed paths.

Averaging methods. Energy balance method for limit cycles. Amplitude and frequency estimates. Nearly-periodic solutions. Periodic solutions and Harmonic balance method.

Perturbation methods for Duffing;s equation. Peridic solution of autonomous systems. Lindstedst's method. Perturbation using two time-scales.

The existence of periodic solutions. The Poincare - Bendixson theorem.

Simple bifurcations. The saddle-node, transcritical and pitchfork bifurcation. Hopf bifurcation.

Manifolds. Stable Manifold and Centre manifold theorem.

Course Outcomes:

On the completion of this course

- *students will be able to study the nature of linear stability and general stability of critical points and solutions of physical /biological systems.*
- *investigate the existence of periodic solutions in physical systems.*
- *have a basic idea about bifurcations of solutions and shifts in stability.*
- *use perturbation method to study non-linear differential equations.*
- *Apply these methods to study problems of population biology /ecology and nonlinear wave propagation.*

References :

1. *Nonlinear Ordinary Differential Equations* - D. W. Jordan, P. Smith, OUP, 2007.

2. *Nonlinear Differential Equations and Dynamical Systems* - F. Verhulst, Springer.
3. *Nonlinear Dynamics and Chaos* - Steven H Strogatz. Levant Books.Kolkata 2007.
4. *An Introduction to dynamical systems* - D. K. Arrowsmith and C. M. Place. CUP 1990.
5. *Differential Equations and Dynamical Systems* - L.Perko. Springer, NY, 1991.
6. *Nonlinear Oscillations, Dynamical systems, and bifurcation of vector fields* - J. Guckenheimer and P. Holmes. Springer NY, 1983.
7. *Nonlinear Ordinary Differential Equations* - R Grimshaw, Blackwell, Oxford, 1990.
8. *Nonlinear Differential Equations* Raimond A Strubble, International Series in pure and Applied Mathematics .
9. *Nonlinear Differential Equation* , P de Muttoni , L . Salvadori , Academic Press, 2014

Course : MTM2P COR 13T

Advanced Numerical Analysis and Partial Differential Equations : 50 Marks (4 CP)

Syllabus :

Advanced Numerical Analysis

Solution of non linear equations, Modified Newton-Raphson method, Aitken's delta square method, Bairstow method, Graeffe's root squaring method and their convergences. System of non linear equations, Newton-Raphson method.

Numerical Solution of linear systems, Matrix inversion method, Error analysis for direct methods, Partial pivoting, Complete pivoting, Operation counts, Successive-Over Relaxation (SOR) iteration method, Convergence, Well and ill condition systems.

Existence and uniqueness of polynomial interpolations, Piecewise polynomial interpolation, Convergence properties, Least square polynomial approximations, Approximations using orthogonal polynomials, Chebyshev polynomials.

Gauss-Legendre and Gauss-Chebyshev quadratures, EulerMaclaurin summation formula, Richardson extrapolation, Romberg integration, Improper integrals.

Numerical Solutions of Boundary Value Problems, Finite difference method, Solution of linear and non-linear equations by the Shooting method, Introduction to finite element methods.

Partial Differential Equations

Origin of 2^{nd} order PDEs, Fundamental equations of Physics.

Laplace equation, Dirichlet and Neumann problems (including three dimensional cases) , Solution using Green's function method, Maximum and minimum principles, existence and uniqueness of solutions.

Riemann's method for solving hyperbolic PDEs, the problem of vibrations of a membrane, non-homogeneous problems, Duhamel's principle.

BVPs for the diffusion equation, Solution using Green's functions.

Notion of non linearity, diffusion and dispersion processes, Burger and KdV equations.

Course Outcomes: After completion of the course, the student is expected to :

- understand basic theories of numerical analysis,
- formulate and solve numerically problems from different branches of science,
- grow insight on computational procedures.
- learn to solve different types of PDE,
- test the stability of the solution,
- apply PDE to problems of geometry and physics
- formulate and solve problems from allied branches of science.

References:

1. S.D. Conte and C. DeBoor, Elementary Numerical Analysis: An Algorithmic Approach, McGraw Hill, N.Y., 1980.

2. A. Ralston, A First Course in Numerical Analysis, McGraw Hill, N.Y. , 1965.

3. A. Ralston and P. Rabinowitz, A First Course in Numerical Analysis, McGraw Hill, N.Y., 1978.

4. K.E. Atkinson, An Introduction to Numerical Analysis, John Wiley and Sons, 1989.

5. W.F. Ames, Numerical Methods for PDEs, Academic Press, N.Y., 1977.

6. L. Colatz, Functional Analysis and Numerical Mathematics, Academic Press, N.Y., 1966.

7. Jain, Iyengar and Jain, Numerical methods for scientific and Engineering Computation, New Age International Pub.

9. F.B.Hilderbrand, Introduction to Numerical Analysis, Dover Publication.

8. Powell, M. – Approximation Theory and Methods.

9. Sneddon I.N. : Elements of Partial Differential Equations, Mcgraw Hill.

10. Petrovsky I.G. : Lectures on Partial differential equations.

11.. Courant and Hilbert : Methods of Mathematical Physics, Vol – II.

12. L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society.

13. F. John, Partial Differential Equations, Narosa.

14. Williams W.E. : Partial Differential Equations.

15. Miller F.H. : Partial Differential Equations.

16. K.S. Rao, Introduction to partial differential equations, Prentice Hall, New Delhi, 1997.

17. T. Amaranath, An elementary course in partial differential equations, Narosa, 2014

Course : MTM2P DSE 02T

(Elective Paper II*)

1. Continuum Mechanics II: 50 Marks (4 CP)

Syllabus :

Introduction: Fluid Properties. Ideal Fluids. Viscous compressible and incompressible fluids. Non-Newtonian fluids.

Motion of fluid: Lagrangian and Eulerian methods, Equation of continuity in different coordinate systems, Boundary surfaces, Stream lines, Path lines and streak lines. Velocity potential, Irrotational and Rotational motions. vortex lines. Euler's equation of motion, Bernoulli's equation and applications to some special cases, Helmholtz's equation for vorticity, Impulsive generation of motion and some properties,

Two dimensional irrotational flow, Velocity potential, Circulation, Kelvin's circulation theorem, Kelvin's theorem of minimum kinetic energy. Stream function, Complex potential.
Three-dimensional motion, Source, Sink, Doublet, Complex potential and images with respect to straight line and Circle, Milne Thomson circle theorem, Blasius theorem.

Navier-Stokes' Equations, Boundary Conditions.

Viscous incompressible fluid flow: Similarity of Flows, Reynold's number, Vorticity equation. Circulation, Flow through parallel plates, Couette flow, Plane Poiseuille flow, Flow through pipes of circular and elliptic cross sections.

Inviscid Compressible Fluid: Field equations, Circulation, Propagation of small disturbance. Mach number and cone, Bernoulli's equation. Irrotational motion, Velocity potential. Bernoulli's equation in terms of Mach number. Pressure, density, temperature in terms of Mach number, Critical conditions. Steady channel flow, Area-velocity relation. Mass flow through a converging nozzle. Flow through a de-Laval nozzle. Normal shock waves. Surface waves, Progressive waves, Group velocity, Standing waves.

Course Outcomes: After completing this course, the student will be able to:

- understand the concept of fluids, their classification, flow lines, as well as the Eularian and Lagrangian descriptions of fluid motion, and categories fluids and flows based on their physical characteristics.
- derive and solve the equation of continuity, equations of motion, equation of vorticity.
- know velocity potential, stream function, and complex potential, Bernoulli's equation, Kelvin's minimum energy and circulation theorems.
- understand two- and three-dimensional motion, circle theorem, Blasius theorem
- understand flow through parallel plates, Couette flow, plane Poiseuille flow, Mach number, mass flow through a converging nozzle, normal shock waves, surface waves.

References :

1. G. K. Batchelor. An Introduction of Fluid Mechanics, Cambridge University Press (2000).
2. M. Rahman, Mechanics of Real Fluids, WIT Press (2011).
3. P. K. Kundu, I. M. Cohen & D. R. Dowling, Fluid Mechanics, Academic Press (2016).
4. M. D. Raisinghania, Fluid Dynamics, S. Chand and Co. Ltd. (2003).
5. F. Chorlton, Textbook of Fluid Dynamics, G. K. Publishers (2012).
6. J. L. Bansal - Viscous Fluid Dynamics, Oxford and IBH Publishing Co. (1977).

2. Number Theory and Equations over Finite Fields: 50 Marks (4 CP)

Syllabus :

Prime Numbers and Unique Factorization, Primes in Arithmetic Progressions, Euclids Alghorithm, Wilsons Theorem, Linear congruence; ax ≡ b(mod n), Sums of Two Squares, Chinese Remainder Theorem, Euler's Theorem.

Primitive roots modulo n, the existence of primitive roots, applications of primitive roots, Structure of U(Z/nZ), The equation $x^n \equiv a \pmod m$ (n th Power residues), The ring of Gaussian integers Z[i], Integral Binary Quadratic forms $aX^2 + bXY + cY^2$,

Quadratic Reciprocity Laws: Legender Symbol and a Gauss Sum, proof of the law of quadratic reciprocity.

Equations over Finite Fields : Finite Fields, Gauss and Jacobi Sums,: Chevalley-Warning Theorem, Quadratic Forms over finite fields and their reduction to the equation $a_1 x^{l_1} + a_2 x^{l_2} + \cdots + a_r x^{l_r}$

$= b$ over F_q.

Construction of p-adic numbers, ring of p-adic integers, some appliations.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Wilsons Theorem, Linear congruence; ax ≡ b(mod n),
ii) Chinese Remainder Theorem, Euler's Theorem,
iii) applications of primitive roots, Structure of U(Z/nZ),
iv) law of quadratic reciprocity,
v) Equations over Finite Fields: Chevalley-Warning Theorem,
vi) Quadratic Forms over finite fields,
vii) p-adic numbers and its applications.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Number Theory and Equations over Finite Fields, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions.

References :

1. D. M. Burton; Elementary Number Theory; Wm. C. Brown Publishers, Dulreque, Lowa, 1989
2. Stilwell, J., Elements of Number Theory, Sprinter UTM 2003.
3. Gareth A. Jones and Mary Jones J., Elementary Number Theory, Springer SUMS 2005.
4. Neal Koblitz; A course in number theory and cryptography; Springer-Verlag, 2nd edition.
5. Ireland K. and Rosen M., A Classical Introduction to Modern Number Theory, Springer GTM 2004.
6. Lidl R. and Niedrreiter H., Finite Fields, Encyclopedia of Mathematics and its Applications 20, Cambridge 1997.
7. Richard A Mollin; Advanced Number Theory with Applications; CRC Press, A Chapman & Hall Book.
8. Flath D.E., Introduction to Number Theory, John Wiley & Sons 1989.

3. Fuzzy sets & Their applications: 50 Marks (4 CP)

Unit: 1

Fuzzy sets-Basic definitions. Level sets, Convex fuzzy sets. Basic operations on fuzzy sets.Types of fuzzy sets. Cartesian products. Algebraic products bounded sum and difference f norms and t-co norms.

Unit: 2

The Extension principle-The Zadeh's extension principle image and inverse image of fuzzy sets

Unit: 3

Fuzzy numbers. Elements of fuzzy arithmetic., Fuzzy Relations and Fuzzy Graphs-fuzzyrelations on fuzzy sets. Composition of fuzzy relations, Min-Max composition, and itsproperties.

Unit: 4

Fuzzy compatibility relations Fuzzy relation equations. Fuzzy graphs. Similarity relation.

Unit : 5

Fuzzy Logic- An overview of classical logics. Multivalued logics. Fuzzy. Propositions.
Fuzzy quantifiers. Linguistic variables and hedges.

Unit: 6

Possibility Theory-Fuzzy measures. Evidence theory, Necessity; measure. Possibility theory versus probability theory.

Unit : 7

Decision Making in Fuzzy Environment -individual decision-making. Multiperson decision making. Multicriteria decision-making. Multistage decision making fuzzy ranking methods.Fuzzy linear programming.

Course Outcomes: After completing this course, the student will be able to:

- understand basic knowledge of Fuzzy sets and Fuzzy logic,
- apply basic Fuzzy inference and approximate reasoning,
- apply basic Fuzzy system modeling methods.

References :

1. George J Klir and Tina A Folger, Fuzzy sets-Uncertainty and Information, Prentice Hall of India, 1988.
2. H. J. Zimmerman, Fuzzy Set theory and its Applications, 4th Edition, Kluwer Academic Publishers, 2001.
3. George J Klir and Bo Yuan, Fuzzy sets and Fuzzy logic: Theory and Applications, Prentice Hall of India, 1997.
4. Timothy J Ross, Fuzzy Logic with Engineering Applications, McGraw Hill International Editions, 1997.
5. Hung T Nguyen and Elbert A Walker: A First Course in Fuzzy Logic, 2nd Edition Chapman & Hall/CRC 1999.
6. Jerry M Mendel, Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions, PH PTR, 2000.
7. John Yen and Reza Langari, Fuzzy Logic: Intelligence, Control and Information, Pearson Education, 1999.

Course : MTM2P SEC 01M

Mathematical Computing with Mathemtica/Matlab &Introduction to LATEX : 50 Marks (2 CP)

Syllabus :

Introduction, Numerical and symbolic computations, Lists, strings, rules, patterns and pattern matching. Working with matrices (addition, subtraction and multiplication), creation of tables, plotting 2D, 3D graphs etc. Linear and polynomial algebra.

Basic Programs to be written in Mathematica/Matlab on Conditional Control constructs, Loops, User defined functions and Library functions, Arrays (Single and Multi-dimensional), Structures etc.

Mathematica/Matlab programs on Numerical methods for finding solution of algebraic and Transcendental equations, System of linear equations, Interpolation, Numerical differentiation & Numerical integration and Numerical solutions of ordinary and partial differential equations.

Latex: Introduction to Latex. Document structure. Typesetting text, math formulas and expressions. Tables, Figures, Equations. Bibtex, Beamer presentation.

Course Outcomes : After completing this course, the student will be able to:

- understand the concept of different types of Variables, Functions, Operators, and Data types.
- perform Input and Output operations.
- understand graphics and plotting Graphs.
- understand basic features of programming in MATHEMATCA/MATLAB.
- learn typesetting math in LaTeX, adding a picture, generating a table of contents, adding bibliography, adding footnotes, creating tables with LaTeX
- to create scientific documents for project/ dissertation and professional presentations.

References:

1. Eugene Don, Mathematica 2nd edition, Schaums Outlines, McGraw-Hill (2009).

2. Paul Wellin, Programming with Mathematica : An Introduction, Cambridge University Press (2013).

3. Rudra Pratap, Getting Started with Matlab, A quick introduction for Scientists and Engineers, Oxford University Press (2019).

4. L. Lamport, LaTeX: a Document Preparation System, 2nd Edition, Addison-Wesley (1994).

Semester : IV

Course : MTM2P DSE 03T
(Advanced Paper I**)
1. Advanced Functional Analysis: 50 Marks (4 CP)

Syllabus :
Convex sets, convex hull, Representation Theorem for convex hull. Symmetric sets, balanced sets, absorbing sets and their properties, absolutely convex sets. Topological vector spaces, homeomorphisms, local base, locally convex topological vector spaces, bounded sets, totally bounded sets, connectedness and their basic properties. Separation properties of a topological vector space, compact and locally compact topological vector space and its properties on finite dimensional topological vector spaces, convergence of filter, completeness, Frechet space, quotient spaces, separation property by hyperplane on locally convex topological vector spaces, Linear operators over topological vector space, Boundedness and continuity of linear operators, Minkowski functionals and its basic properties, Hyperplanes, Separation of convex sets by Hyperplanes, Extreme points, Krein-Milman Theorem on extreme points, Metrizability of topological vector spaces.

Geometric form of Hahn Banach Theorem. Uniform boundedness principle, open mapping theorem and closed graph theorem for Frechet spaces. Banach-Alaoglu theorem.

Seminorms and its properties, Generating family of seminorms in locally convex topological vector spaces, Criterion for normability of a topological vector space (Kolmogorov Theorem).

Weierstrass Approximation Theorem in C[a,b], best approximation theory in normed linear spaces, uniqueness criterion for best approximation. Separable Hilbert Space, Strict convexity and uniform convexity of a Banach space with examples, Uniform approximation, Haar condition, Haar uniqueness theorem.

Only statements of Clarkson's Renorming Lemma and Milman and Pettit's theorem, Uniform convexity of a Hilbert space, Reflexivity of a uniformly convex Banach space.

Course Outcomes: Upon successful completion, students will have the knowledge and skills to explain the fundamental concepts of functional analysis and their role in modern mathematics and applied contexts. Moreover, students will be able to demonstrate accurate and efficient use of functional analysis techniques.

References:

1. W. Rudin, *Functional Analysis,* TMG Publishing Co. Ltd., New Delhi, 1973**.**
2. E. Kreyszig, *Introductory Functional Analysis with Applications,* Wiley Eastern, 1989.
3. G. Bachman and L. Narici, *Functional Analysis,* Academic Press, 1966.
4. A. E. Taylor- *Functional Analysis*, John Wiley and Sons, New York, 1958.
5. L. Narici & E. Beckenstein, *Topological Vector spaces*, Marcel Dekker Inc, New York and Basel,1985.
6. A. A. Schaffer, *Topological Vector Spaces,* Springer, 2nd Edn., 1991.
7. J. Horvoth, *Topological Vector spaces and Distributions,* Addison-Wesley Publishing Co., 1966.

2. Advanced Real Analysis: 50 Marks (4 CP)

Syllabus :

Representation of real numbers by series of radix fractions. Sets of real numbers,Derivatives of a set. Points of condensation of a set. Structure of a bounded closed set.

Perfect sets. Perfect kernel of a closed set. Cantor's nondense perfect set. Sets of first and second categories, residual sets. Baire one functions and their basic properties. One-sided upper and lower limits ofa function. Semicontinuous functions. Dini derivates of a function. Zygmund's monotonicity criterion.

Vitali's covering theorem. Differentiability of monotone functions and of functions of bounded variation. Absolutely continuous functions, Lusin's condition,characterization of AC functions in terms of VB functions and Lusin's condition. Concepts of VB*, AC*,VBG*, ACG* etc. functions. Characterization of indefinite Lebesgue integral as an absolutely continuous function.

Generalized Integrals**:** Gauge function. Cousin's lemma. Role of gauge functionin elementary real analysis. Definition of the Henstock integral and its fundamental

properties. Reconstruction of primitive function. Cauchy criterion for Henstock integrability. Saks-Henstock Lemma. The Absolute Henstock Integral. The McShane integral. Equivalence of the McShane integral, the absolute Henstock integral and the Lebesgue integral. Monotone and Dominated convergence theorems. The Controlledconvergence theorem.

Definition and elementary properties of the Perron integral and its equivalence with the Henstock integral. Definition of the (special) Denjoy integral and its equivalence with the Henstock integral (characterization of indefinite Henstock integral as a continuous ACG*function). Density of arbitrary sets. Approximate continuity. Approximate derivative.

Course Outcomes: After completing the course, the students should be able to recognize, understand and apply concepts and methods in advanced real analysis. Also, they will be able to apply the acquired knowledge in signals and Systems, Digital Signal Processing etc. and conduct researches on high international level in advanced real analysis.

References :

1. E. W. Hobson,The Theory of Functions of a Real Variable (Vol. I and II).

2. I. P. Natanson,Theory of Functions of a Real Variable (Vol. I and II).

3. R. Henstock,Lectures on the Theory of Integration.

4. E. J. McShane,Unified Integration.

5. S. Saks,Theory of the Integral.

3. Advanced Complex Analysis: 50 Marks (4 CP)

Syllabus :

Elementary properties of holomorphic functions: Basic properties of holomorphic functions, relations with the fundamental group and covering spaces; the Open Mapping Theorem;

The Maximum Modulus Principle : The Schwarz Lemma, The Phragmen- Lindeloff Method, a converse of Maximum Modulus Theorem.

Approximation by Rational functions: Runge's Theorem, simply connected regions, the Mittag-Leffler's theorem for Meromorphic function.

Zeros of holomorphic functions: Infinite products, the Weierstrass Factorization Theorem, Jensen's formula, The Muntz-Szasz theorem.

Analytic Continuation : Direct analytic continuations, uniqueness of analytic continuation along a curve, Monodromy theorem and its consequence, the Little Picard Theorem.

Conformal mapping : Normal families, the Riemann mapping Theorem.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Basic properties of holomorphic functions,
ii) The Phragmen- Lindeloff Method, a converse of Maximum Modulus Theorem,
iii) the Mittag-Leffler's theorem for Meromorphic function,
iv) the Weierstrass Factorization Theorem, Jensen's formula, The Muntz-Szasz theorem,
v) Monodromy theorem and its consequence, the Little Picard Theorem,
vi) the Riemann mapping Theorem,
vii) multilinear algebra, differential forms, the Lie derivative..

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Advanced Complex Analysis, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions.

References:

1. J.B. Conway, Functions of one complex variables Springer - Verlag, International student Edition, Narosa Publishing Co.
2. L. Hahn, B. Epstein, Classical Complex Analysis, Jones and Bartlett, India, New Delhi, 2011
3. W. Rudin, Functional analysis.
4. S. Lang, Real analysis.
5. L.V. Ahlfors, Complex Analysis, Mc. Graw Hill Co., New York, 1988.
6. W. Rudin, Real and complex analysis, McGraw-Hill, 1987.

4. Advanced Topology I: 50 Marks (4 CP)

Syllabus :

Nets and Filters : Inadequacy of sequences. Nets & filters. Topology and convergence of nets & filters. Subnets. Ultranets & Ultra filters. Canonical way of converting nets to filters and vice-versa. Chracterizations of compactness and continuity and adherent point in terms of nets and filters. Convergence of nets and filters in product spaces.

Local Compactness and One Point Compactification.

Stone- Cech Compactification. Extension property of Stone- Cech Compactification βX . Cardinality of βN .

Embedding and Metrization. Embedding Lemma and Tychonoff Embedding. The Urysohn Metrization Theorem. The Nagata – Smirnov Metrzation Theorem (statement only).

Paracompacntess : Different types of refinements and their relationships. Paracompacntess – definition in terms of locally finite refinement, various characterizations of Paracompactness in regular spaces. A. H. Stone’s Theorem concerning paracompactness of metric spaces. Partition of unity and Paracompacntess. Properties of Paracompacntess w.r.to subspaces and products.

Uniform spaces : Definition and examples. Base and subbase of a uniformity . Uniform topology, uniform continuity and product uniformity. Uniformization of topological spaces. Uniform property. Uniformity generated by a family of pseuometrics. Cauchy filter. Completeness of uniform spaces. Completion of uniform spaces. Compactness and uniformity. Uniform cover.

If time Permits:
Inductive and projective limits: Inductive and projective limits of topological spaces.
Function spaces.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge on the following :

i) Inadequacy of sequences, Nets and Filters , Chracterizations of compactness and continuity and adherent point in terms of nets and filters,
ii) Local Compactness and One Point Compactification, Stone- Cech Compactification, Extension property of βX and Cardinality of βN,
iii) The Urysohn Metrization Theorem. The Nagata – Smirnov Metrzation Theorem,
iv) Paracompacntess, Partition of unity, A. H. Stone’s Theorem,
v) Uniform spaces and Uniform topology, uniform continuity and product uniformity, Uniformity generated by a family of pseuometrics, Completion of uniform spaces,
vi) Inductive and projective limits, Function spaces.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Advanced Topology I, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. General Topology by J. L. Kelley, Van Nostrand.
2. General Topology by S. Willard, Addison-Wesley.
3. Topology by J. Dugundji, Allyn and Bacon.
4. Topology, A first course by J. Munkres, Prentice Hall, India.
5. Introduction to topology and modern analysis by G. F. Simmons, McGraw Hill.
6. Introduction to General topology by K. D. Joshi, Wiley Eastern Ltd.
7. General Topology by Engelking, Polish Scientific Publishers, Warszawa.
8. Counter examples in Topology by L. Steen and J. Seebach.

Course : MTM2P DSE 04T
(Advanced Paper II**)

1. Mathematical Biology: 50 Marks (4 CP)

Syllabus :

1. Deterministic models of population growth .Continuous growth models. Logistic growth law. Allee effect. Bacterial growth. Harvesting. Functional responses. The spruce budworm population. Models of interacting populations. The Lotka-Volterra model for competition. The Lotka Volterra predator – prey model. Gause –Model , Kolmogorov Model , Leslie-Gower Model .Models with Ratio-dependent functional responses .Complexity and stability in a generalised predator-prey system. Predator-prey models with logistic growth in prey and Holling-type responses. Analysis of such models with limit cycle periodic behaviour. Mutualism. Host parasite model. General food-chain models. Nutrient-Phytoplankton-Zooplankton interaction.

2. Biological mechanisms responsible for "time-delay". Discrete and continuous time-delay. The single species logistic model with the effect of time-delay. Stability of equilibrium position for the logistic model with general delay function. Stability of logistic model for discrete time lag. Time-delayed H-P model together with their stability analysis. Prey-predator Model with time-delay.

3. Spatial population models. Metapopulations. Reaction-diffusion model. Biological waves. Single – species model. Fisher-Kolmogoroff equation and travelling wave solutions.

4.Models of Epidemics.

Introduction; Some basic definitions. Simple epidemic model, General epidemic model. Kermack-McKendrik threshold theorem. Recurring epidemic model. A comparative study of these models. Control of an epidemic.. Models having multiple infections. Epidemic model with multiple infections. Stochastic epidemic model with removal. Stochastic epidemic model with removal, immigration and emigration. Special discussion on the stochastic epidemic model with carriers. Simple extensions of SIR model: Different case studies --- (i) Loss of immunity, (ii) Inclusion of immigration and emigration, (iii) Immunization. SIR endemic disease model.

Course Outcomes:

- After completion of this course, students should be able to formulate realistic mathematical models for diverse biological phenomena and analyse them mathematically to explain the observations as obtained from experiments, clinical trials and observations.
- Students would learn to mathematically predict the outcome in a situation by constructing and theoretically analysing a model.
- The students will learn how to develop mathematical models which provide ways to design and evaluate protocols to manage and control animal populations, natural resources like forests, wildlife, fisheries, and outbreak of diseases.

References :

1.H I Friedman -- Deterministic Mathematical Models in Population Ecology ,Marcel Dekker ,(1980)

2 J. D. Murray – Mathematical Biology 1. An Introduction., Springer-Verlag, Berlin (2002).

3 R. M. Andersson and R M May-- Infectious Diseases of Humans : Dynamics and control, OUP, (1991).

4. J. N. Kapur --- Mathematical Models in Biology and Medicine, East West Press Pvt Ltd (1985)

5. 6. R. W. Poole --- An Introduction to Quantitative Ecology, McGraw- Hill, (1974).

6. E. C. Pielou -- An Introduction to Mathematical Ecology, Wiley, New York, (1977).

7. R. Rosen -- Foundation of Mathematical Biology (vol. I& II), Academic Press, (1972).

8. R. M. May --- Stability and Complexity in model ecosystems ,Princeton University Press, (2001).

9. Mark Kot – Elements of Mathematical Ecology, Cambridge University Press, (2003).

10. J. M. Smith-- Mathematical Ideas in Biology. CUP, (1968).

11. L. J. S. Allen-- An introduction to Mathematical Biology, Pearson/Prentice Hall, (2007).

2. Solid Mechanics: 50 Marks (4 CP)

Syllabus :

Formulation of Problems in Elasticity: Review of field equations. Boundary conditions and fundamental problem classifications. Stress and displacement formulation. ‘Uniqueness of solutions. Clapeyron’s Theorem. Saint-Venants principle.

Problems in Elastostatics: Plane strain. Plane stress. Boundary conditions. Airy’s stress function. Biharmonic boundary value problems. The first and second boundary value problems. Existence and uniqueness of solutions. Conformal maps. Simply connected domains. Solution of basic problem in a circular region.

Extension, Torsion and Flexure of Beams : Statement of Problem. Extension by longitudinal forces. Beam stretched by its own weight. Bending by terminal couples. Torsion of circular shaft. Torsion of cylindrical bars. Torsion function. Neumann's problem. Stress function. Dirichlet's problem. Flexure of beams by terminal loads. Neumann and Dirichlet's problems. Centre of flexure. Bending by a load along the principal axis. Bending of rectangular beams.

Thermo-elasticity : Thermal stress. Stress-strain relation in Thermo-elasticity.

Problems in Elastodynamics: Theory of elastic waves; Motion of a surface of discontinuity – kinematical condition and dynamical conditions. Kirchoff's solution of inhomogeneous wave equation. Reflection and refraction of elastic body waves.

Surface waves : Rayleigh, and Love waves.

Course Outcomes:

- *This course is intended to give the students an introduction to different types of problems related to static deformation and dynamical motions arising in the Theory of linear Elasticity.*
- *The students will be able to analyse mechanical problems related to bending , torsion and flexure of beams.*
- *The students will be able to study analytically acoustic disturbances moving through solids.*
- *On completion of this course students will have learnt the fundamental concepts required for research in Applied Mechanics .*
- *The students will learn the basic ideas of thermal deformations and coupled thermoelasticity.*

References :

2. I. S. Sokolnikoff: Mathematical Theory of Elasticity. McGraw Hill, 1956.
3. 2. A. E. H. Love: A treatise on Mathematical theory of Elasticity. Dover, 1954.
4. P.L. Gould: Introduction to linear elasticity. Springer-Verlog, 1994.
5. N. I. Muskhelisvili: Some basic problems on the theory of elasticity. Nordhoff, 1953.
6. Y. C. Fung: Foundation of solid mechanics. Prentice Hall, 1965.
7. L. D. Landau and E. M. Lifshitz: Theory of Elasticity. Pergamon Press, 1989.
8. S. Timoshenko and S. N. Goodier: Theory of Elasticity. McGraw Hill, 1970.
9. V. Z. Parton and P. I. Perlin: Mathematical Methods of the theory of elasticity. vol. I, II, Mir Publishers, 1984.
10. Elastodynamics, Volume II, A. C. Eringen and E. S. Suhubi, Academic Press, 1970.
11. Theory of Elasticity Y A Amenzade, MIR Publishers
12. Some Basic Problems of the Mathematical Theory of Elasticity , N I Mushkelishvili.
13. Thermoelasticity W Nowacki
14. Wave Motion in Elastic Solids , Karl F Graff , (Dover books on Physics), 1991

3. Advanced Operations Research: 50 Marks (4 CP)

Syllabus :

Network Analysis: Network definitions, Minimal Spanning Tree Algorithm, Shortest Route Algorithms, Max-flow Min-cut theorem, Genaralized Max-flow Min-cut theorem, linear programming interpretation of Max-flow Min-cut theorem, minimum cost flows. A brief introduction to PERT and CPM, Components of PERT/CPM Network and precedence relationships, Critical path analysis, PERT analysis in controlling project.

Optimal Control Theory: Performance criterion, Unconstrained systems, Application of calculus of variation, constrained systems, Pontryagin's principle, Quadratic performance criterion, Regulator problem.

Replacement Models: Types of replacement problems, replacement of items deteriorates with time, Replacement policy for equipments when value of money changes with constant rate during the period, Replacement of low cost items, Group replacement, Individual replacement policy, Mortality theorem, Recruitment and promotional problems.

Matrix Game: Definition of a non-cooperative game. Admissible situation and the equilibrium situation, strategic equivalence of games. Antagonistic Games, Saddle points. Matrix Games. Mixed strategies. Existence of minimaxes in mixed strategies. Convex sets. The value of the game and optimal strategies.

Continuous Games: Continuous games on unit square. Continuous game. Equilibrium Situation. Fundamental Theorem. Devices for Computing and Verifying Solutions.

Differentiable Game: Two person deterministic continuous differential games, Two person zero-sum differential games, Pursuit games, Co-ordination differential games, Noncooperative differential game.

Simulation: Basic concepts, Monte Carlo method, Random number generation, Waiting the simulation model, New process planning through simulation, Capital budgeting through simulation.

Course Outcomes: Upon completion of this course, the student will be able to**:**

- formulate operation research models to solve real life problem,
- understand the mathematical tools that are needed to solve optimization problems,
- describe Optimal Control Theory and their applications,
- analyze game theory,
- understand skills and knowledge of operations research and its application in industry.

References :

1. Joseph J. Madder, Cecil R, Philips, Project Management with PERT and CPM.
2. Panel A. Jensen, Wesley Barness J., Network flow programming, John Wiley and Sons, 1980.
3. OR methods and Problems - Sasieni Maurice, Arther Yaspan, Lawrence Friedman.
4. Elmagharby Salah E., Activity Network Project Planning and Control by Network Models, John Wiley and Sons.
5. Operations Research – H. A. Taha.
6. C. Mohan and K. Deep, Optimization Techniques, New Age Science, 2009.
7. Operations Resarch - T.L. Satty.

4. Algebraic Topology: 50 Marks (4 CP)

Syllabus :

Homotopy and Homotopy classes. Homotopy equivalences, Null homotopy, Relative homotopy, Composite of homotopic spaces.

Contractible spaces, deformation, strong deformation retraction, Path-connected spaces - their union, intersection and continuous images.
Product and inverse of paths. Homotopy of paths and products of homotopic paths.

Covering spaces and covering maps. Properties of covering maps. Path lifting property and Homotopy lifting theorem.

Fundamental group : Definition and verification. Homomorphism and isomorphism of fundamental groups. Fundamental groups of Circle. Fundamental groups of some known surfaces - Cylinder, punctured plane, Torus, etc.

Finite Simplicial Complexes : Simplicial complexes. Polyhedra and Triangulation.
Simplicial approximation, barycentric subdivision and simplicial approximation theorem.

Simplicial Homology : Orientation of simplicial complexes. Simplicial chain complexes, boundaries and cycles, homology groups – some examples. Induced homomorphisms. Reduced homology groups.
Some applications, e.g., Invariance of dimension, no-retraction theorem, Brower's fixed point theorem, etc.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Homotopy, Contractible spaces, deformation, strong deformation retraction,
ii) Covering spaces and covering maps,Path lifting property and Homotopy lifting,
iii) Fundamental groups of Circle, Cylinder, punctured plane, Torus, etc.,
iv) Simplicial complexes. Polyhedra and Triangulation, barycentric subdivision and simplicial approximation theorem,
v) Simplicial Homology, homology groups, no-retraction theorem, Brower's fixed point theorem.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of algebraic topology to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. Algebraic Topology, A. Hatcher, Cambridge University Press.
2. Algebraic Topology, W.S. Massey, Springer (GTM).
3. Algebraic Topology : A first course, M.J. Greenberg and J. R. Harper, Perseus books, Cambridge.
4. Algebraic Topology : A Primer, S. Deo, Hindustan Book Agency (trim 27).
5. Categories for the Working Mathematicians (second edition), S. MacLane, Springer (GTM).
6. Algebraic Topology : A First Course, Springer-Verlag,1995.
7. Essential Topology, Springer ,Martin D. Crossley.
8. Algebraic Topology, J . Munkres.

Course : MTM2P DSE 05T

(Advanced Paper III**)

1. Plasma Dynamics: 50 Marks (4 CP)

Syllabus :

Basic properties of plasmas : Definition of Plasma as an ionized gas. Thermal ionization, Saha equation, Basic defining properties of plasma, Debye shielding, Plasma parameters, plasma frequency, Collisions. Natural occurrence of Plasma. Applications of plasma physics.

Motion of Charged Particle: Motion of charged particles in electric and magnetic fields: Larmor orbits, Particle drifts: $\mathbf{E}\times\mathbf{B}$ drift, polarization drift, curvature drifts, grad B drifts. Magnetic moments, Adiabatic invariants. Concept of Ponderomotive force. Magnetic mirror (concept of plasma confinement).

Plasma kinetic theory: Vlasov equation: Equilibrium solutions, Electrostatic Waves. Concept of Landau damping.

Plasma Fluid Theory: Derivation of fluid equations from the Vlasov equation. Plasma oscillations, Langmuir waves, Dielectric Function, ion-acoustic waves, Electromagnetic waves. Upper and lower hybrid waves, Alfven waves, Ion and Electron cyclotron waves.

Nonlinear Plasma Theory: Concept of nonlinearity and dispersion. Korteweg-de Vries equation: ion acoustic solitary wave and its solution. Nonlinear Schrödinger equation and Envelope soliton.

Course Outcomes: At the end of this course a student should be able to :

- understand collective nature of plasma dynamics by developing concepts of Debye screening collective behavior and quasi neutrality,
- describe motion of charged particles in electric and magnetic fields,
- derive the basic set of fluid equations to study plasma properties,
- know the concept of Landau damping,
- describe the propagation of waves in plasmas and understand the concept of nonlinearity and dispersion relation.

References :

1. F. F. Chen - Introduction to Plasma Physics, Plenum Press, New York and London (1977).
2. D. R. Nicholson - Introduction to Plasma Theory, John Wiley and Sons, NY, (1983).
3. T. J. M. Boyd and J. J. Sanderson - The Physics of Plasmas, Cambridge University Press, (2003).
5. R. C. Davidson - Methods in Nonlinear Plasma Theory, Academic Press, New York and London (1972).
6. J. A. Bittencourt - Fundamentals of Plasma Physics, Springer-Verlag New York, (2008).
7. N. A Krall and A. W. Trievelpiece - Principles of Plasma Physics, McGraw Hill Kogakusha, Ltd., Tokyo, New Delhi etc. (1973).
8. P. C. Clemmow and J. P. Dougherty - Electrodynamics of Particles and Plasma.
9. B. Chakraborty - Principles of Plasma Mechanics.

2. Advanced Fluid Dynamics: 50 Marks (4 CP)

Syllabus :

1. Two and Three dimensional Inviscid incompressible fluid flow : : Field equations; Irrotational motion in simply connected and multiply connected regions. Source, sink,doublet. Image systems. Motion of solid bodies in fluid. Axi-symmetrical motion, Stokes' stream function, Two dimensional motion, Stream function, complex potential, motion of translation and rotation of circular and elliptic cylinders in an infinite liquid, Ciculation. Kelvin's Theorem. Cyclic and acyclic motion. Superposition of motion, circle theorem, Blasius theorem, Kutta Joukowski's theorem.

2. Surface waves, progressive waves in deep and shallow water, Stationary waves, energy and group velocity.

3. Viscous incompressible fluid flow: Similarity, Reynold's number, Flow between parallel plates. Couette and plane Poiseuille flow. Flow through pipes of circular, annular and elliptic cross sections.

4. Laminar Boundary layer.

5. Inviscid compressible flow : Field equations, Circulation, Propagation of small disturbance. Mach number and cone, Bernoulli's equation. Irrotational motion, Velocity potential. Bernoulli's equation in terms of Mach number. Pressure, density, temperature in terms of Mach number, Critical conditions. Steady channel flow, Area-velocity relation. Mass flow through a converging nozzle. Flow through a de-Laval nozzle. Normal shock waves, Governing equations and the solution.

6. Viscous compressible flow: Field equation of compressible flow, Crocco-Vazsonyl equation

Course Outcomes:

1. This course introduces fundamental ideas of fluid dynamics which can be further applied to problems of mechanical engineering.

2. On completion of this course, students would be able to enter research work in Advanced Fluid Theory and Computational Fluid Dynamics (CFD).

References :

1. H. Lamb- Hydrodynamics, Dover Publication.
2. L.M. Milne-Thomson, Theoretical Hydrodynamics.
3. L. Prandtl- Essential of Fluid Dynamics, Hafnen, Pub. Co.
4. P.K. Kundu and Iva M.Cohen-Fluid Dynamics, Har Court, India.
5. J.J. Stoker - Water waves, the mathematical theory with application, Interscience Publ.
6. S.I. Pai- Viscous Flow Theory, Princeton.
7. F. Chorlton- Text Book of Fluid Dynamics, CBS Publ.
8. S.W.Yuan - Foundations of fluid Mechanics, PHI India, 1969.

3. Theory of Waves in Solids: 50 Marks (4 CP)

Syllabus :

1. Elastodynamic theory. Linearized equations of Elasticity. Uniqueness of solutions. Scalar and vector potentials. Wave motion generated by body forces. Point loads. Boundary value problems.
2. Half-space subjected to uniform surface traction. Waves in one-dimensional stress. Harmonic waves.
3. Elastic waves in unbounded medium. Plane waves. Two dimensional wave motion with axial symmetry. Propagation of wave fronts.
4. Plane harmonic waves in elastic half-spaces. P,Sh and SV waves. Reflection. Rayleigh and Stoneley waves.
5. Elastic waves in waveguides.SH waves in a layer.SH modes. Energy Transport . Group velocity and dispersion. Love waves. Lamb waves.
6. Waves in rods and shells. Thin rods. Finite rods. Frequency equation for solid circular rods, Torsional, flexural and longitudinal modes, waves in cylindrical shells.

Course Outcomes:

1. On completion of the course, students will be conversant with propagation of waves in rods, plates and half-spaces.
2. They will be introduced to the basic seismological waves and acoustic waves.
3. The course will be beneficial to students interested in research in applied mechanics or geophysics.

References :

1. Elastodynamics, Volume II, A.C. Eringen and E, S. Suhubi, Academic Press, 1974.

2. Wave Propagation in Elastic Solids , J. D. Achenbach, North –Holland, 1975.

3. Wave motion in elastic solids, Karl F Graff , Dover, 1975.

4. Elastic Waves in layered media, W. M. Ewing, W. S. Jardetzky and F Press, McGraw Hill, 1957.

4. Commutative Algebra: 50 Marks (4 CP)

Syllabus :

Properties of Maximal, prime and primary ideals, radical, nil-radical and Jacobson radical, local ring, Nakayama's lemma, prime spectrum of a ring and Zariski topology, Noetherian and Artinian rings, Hilbert's Nullstellensatz theorem. Finitely generated modules, tensor product of modules, exactness properties of tensor product.

Rings and Modules of fractions, localization and local properties, primary decomposition and associated primes, Integral dependence and independence, integral closure, integrally closed integral domain, Going up and going-down theorems.

Valuation rings, discrete valuation ring, Dedekind's domain, graded rings and modules, completion of modules, Krull intersection theorem. Dimension theory – Dimension theorem of Noetherian local rings.

Course Outcomes: On successful completion of this course, students will be able to apply its methods in related subjects of Mathematics. Moreover, they should be able to participate in scientific discussions and begin with own research in commutative algebra.

References :

1. M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, Addison–Wesley, 1969.
2. N. Bourbaki, *Commutative Algebra*, Hermann, 1972.
3. D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer–Verlag, 1995.
4. I. Kaplansky, *Commutative Rings*, The University of Chicago Press, 1974.
5. E. Kunz, *Introduction to Commutative Algebra and Algebraic Geometry*, Birkh¨auser, 1985.
6. H. Matsumura, *Commutative Algebra*, Benjamin, 1970.
7. H. Matsumura, *Commutative Ring Theory*, Cambridge University Press, 1986.
8. M. Nagata, *Local Rings*, Wiley Interscience, New York, 1962.
9. O. Zariski and P. Samuel, *Commutative Algebra*, Vol. 1, Van Nostrand, 1958.

Course : MTM2P DSE 06T

(Advanced Paper IV**)

1. Quantum Mechanics: 50 Marks (4 CP)

Syllabus :

Experimental background of quantum mechanics; deBroglie waves, Wave-particle duality; Wave functions and Schrodinger equation; Uncertainty relation.

Statistical interpretation of wave functions, expectation values, Ehrenfest's theorem; Time-independent Schrodinger equation; Energy eigenfunction : Discrete and continuous energy eigenvalues; Infinite and finite square well problems: Parity, Simple harmonic oscillator: Algebraic and analytic methods of solution, Dirac delta function potential, free particle: wave packets.

Representation of observables, Dirac's bra-ket notations, mathematical set up on Hilbert space. Equations of motion: Schrodinger picture, Heisenberg picture, Interaction picture.
The Hydrogen atom. Rotation, angular momentum and unitary groups.

Identical particles, Bosons, Fermions; Pauli exclusion principle; Solids: Free electron gas, Band structure. Quantum statistical mechanics: Maxwell-Boltzmann, Fermi-Dirac and Bose-Einstein distributions. Blackbody spectrum.

First and second order perturbations, degenerate perturbation theory. Fine structure of Hydrogen, spinorbit coupling, Zeeman effect.

Variational method: Rayleigh-Ritz variational principle; Hydrogen molecule ion, ground state ofhelium atom.

Relativistic quantum mechanics: Klein-Gordon equation, plane wave solution. Dirac equation, covariant form, charged particle in electromagnetic field, equation of continuity. Dirac hole theory. Spin of the Dirac particle.

Course Outcomes: At the end of this course a student should be able to :

Understand the fundamentals of quantum mechanics,

Create better grasp on different branches of mathematical physics,

Provide an opportunity to recapitulate application of higher pure mathematics,

Open the gateway to modern electronics and nano science.

References :

1. P.A.M. Dirac, The Principles of Quantum Mechanics, Oxford University Press.

2. D. J. Griffiths, Introduction to Quantum Mechanics, Pearson Prentics Hall.

3. B.H. Bransden and C.J. Joachain, Introduction to Quantum Mechanics.

4. L. I. Schiff, Quantum Mechanics, McGraw-Hill, New York, 1968.

5. R. Eisberg, R. Resnick, Quantum Physics of Atoms, Molecule, Solids, Nuclei and Particles, Wiley.

6. A. Das, Lectures on Quantum Mechanics, Hindusthan Book Agency, New Delhi, 2003.

7. E. Merzbacher, Quantum Mechanics,Wiley, New York.

8. C. Cohen-Tannoudji, B. Diu, and F. Laloe, Quantum Mechanics , Wiley- Interscience Publication.

9. R.P. Feynman, The Feynman Lectures on Physics, 3 Vols., Narosa Publ., New Delhi.

10. L.E. Ballentine, Quantum Mechanics, World Sci. Publ., Singapore.

11. T.F.Jordan, Quantum Mechanics in Simple Matrix Form, Dover Publ.

12. M. Chester, Primer of Quantum Mechanics, Dover Publ.

13. J.P. McEvoy and O. Zarate, Introducing Quantum Theory, Icon Books UK, Singapore.

2. Advanced Dynamical Systems and Chaotic Dynamics: 50 Marks (4 CP)

Syllabus :

Nonlinear Systems. Bifurcations and Symmetry breaking. – the origin of Bifurcation Theory. Examples of different types of bifurcations. Transcritical, pitchfork, saddle-node. Centre manifolds. Bifurcation of equilibrium solutions and Hopf bifurcation.

Difference equations.The logistic map. Periodic solutions and their stability.

Introduction to the theory of Chaos. The Lorenz equations and associated maps. Duffing's equation with negative stiffness.One dimensional chaos. The quadratic map. The tent map.Strange attractors.

Bifurcations in one dimensional maps. Period doubling bifurcations. The Feigenbaum number.Two dimensional maps. Bifurcation in two dimensional maps.

Cantor sets. Dimension and fractals.

Hamiltonian systems.Recurrence. Periodic solutions. Invariant torus and chaos.

Course Outcomes: On completion of this course the students would be able to :

1. apply the ideas of dynamical systems theory to understand and explain various complex phenomena of physics and biology,

2. pursue research in complex dynamical systems, mathematical biology , fractal set theory and other related fields.

References :

1. Dynamical systems differential equations, maps and chaotic behavior, D K Arrowsmith and C M Place., Chapman and Hall.
2. Chaotic Dynamics, Baker and Gollub.
3. Nonlinear Systems, P.G. Drazin, CUP 1992.
4. Nonlinear Differential equations and Dynamical Systems, Verhulst.
5. Nonlinear Oscillations, Dynamical systems and bifurcations of vector fields .J Guckenheimer, P Holmes. Springer NY, 1983.
6. Nonlinear Dynamics and Chaos, S.H. Strogatz, Perseus Books, USA, 1994.
7. Differential equations, Dynamical systems and an introduction to chaos, M.W.Hirsch, S.Smale, R.L. Devaney , Academic Press, 2004.
8. Chaos: An Introduction to Dynamical Systems, Kathleen T. Alligood, James A. Yorke, and Tim Sauer Springer NY, 1997.

3. Harmonic Analysis: 50 Marks (4 CP)

Syllabus :

Fourier analysis : Fourier series, pointwise and uniform converges of Fourier series, Fourier transforms, Riemann-Lebesgue lemma, inversion theorem, Parseval identity.
Topological groups: Definition, Basic properties, subgroups, quotient groups, locally compact topological groups, examples. Compact groups: Representations of compact groups, Peter-Weyl theorem, Examples such as SU(2) and SO(3).

Positive Borel measure, Riesz representation theorem, regularity properties of Borel measures.

Haar measure and Haar integral: Invariant measure and Integration, existence and uniqueness of Haar measure and Haar integral on locally compact topological group, Examples of Haar measures Haar Integration.

Elements of Banach algebras: Banach algebra, examples of Banach algebra, algebra with involution, Analytic properties of functions from C to Banach algebras, spectrum and its compactness, commutative Banach algebras, Maximal ideal space, Gelfand topology, Gelfand representation theorem.

Generalization of Fourier transform : Fourier transform on $L_0(G)$ and L(G) (G being a locally compact topological group) Positive definite functions, Bochner characterization, inversion formula, Plancheral theorem, Pontrjagin Duality theorem.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Fourier series, convergence of Fourier series, Riemann-Lebesgue lemma
ii) Basics of Topological groups,
iii) Haar measure and Haar integral,
iv) Banach Algebra and Gelfand topology,
v) Fourier transform on locally compact topological groups,
vi) Plancheral theorem, Pontrjagin Duality theorem.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Harmonic Analysis, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions.

References :

1. Hewitt and Ross; Abstract Harmonic analysis (Vol. I & II); Springer-Verlag, 1963.
2. Bachman, Narici and Beckenstein; Fourier and Wavelet Analysis; Springer.
3. Folland, G. B., A Course in Abstract Harmonic Analysis, CRC Press, 1995.
4. Deitmar, Anton, A First Course in Harmonic Analysis, second edition, Springer, 2002.
5. Walter Rudin; Real and Complex Analysis; McGraw-Hill Book Company, 1921.
6. Katznelson, Yitzhak, An Introduction to Harmonic Analysis, third edition, CUP, 2002.
7. Helson, H., Harmonic Analysis, Addison Wesley, 1983.
8. de Vito, C., Harmonic Analysis - A Gentle Introduction, Jones & Bartlett, 2007.
9. R. R. Goldberg; Fourier Transforms; Cambridge, N.Y., 1961.

4. Advanced Topology II: 50 Marks (4 CP)

Syllabus :

Algebraic Topology :

Covering spaces and covering maps – properties and examples, Path Lifting and Monodromy theorems, Van Kampen's theorem (with a discussion of free and amalgamated products of groups), computing fundamental groups via covering spaces.

Homology - Homology: simplicial homology; singular homology; the Mayer-Vietoris sequence; The Jordan-Brouwer Separation Theorem; the Universal Coefficient Theorem; the Kunneth Formula; CW complexes; cellular homology and computations for projective spaces; the Lefschetz Fixed Point Theorem.

Rings of Continuous Functions :

The ring C(X) & its subring C*(X), their Lattice structure. Ring homomorphism and lattice homomorphism.
Zero-sets, cozero-sets, completely separated sets and its characterization, C-embedding & C* -embedding and their relation, Urysohn's extension theorem . Characterizations of Normal spaces and Pseudocompact spaces in terms of C-embedding & C* -embedding.

Ideals, maximal ideals, prime ideals, Z- ideals; Z-filters, Z- ultra filters, prime filters and their relations. Convergence of Z – filters, cluster points, prime Z – filters and convergence and fixed Z-filters .
Completely regular spaces and the zero-sets, weak topologies determined by C(X) and C*(X). Stone-Čech's therem concerning adequacy of Tychonoff spaces X for investigation of C(X) and C* (X).
Fixed ideals and compactness, fixed maximal ideals of C (X) and C* (X), their characterizations.
Structure spaces.

If time permits :

Topological groups: Basic properties of topological groups, separation properties, subgroups, quotient groups and connected groups.

Course Outcomes: On completion of this course, the students will be able to identify , analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) Covering spaces and covering maps, Path lifting property and Homotopy lifting,
ii) Monodromy theorems, Deck transformation, Van Kampen's theorem,
iii) Singular Homology, Mayer-Vietoris sequence, Idea of Cohomology,
iv) C-embedding & C* -embedding and their relation, Urysohn's extension theorem,
v) maximal ideals, prime ideals, Z- ideals; Z-filters, Z- ultra filters,
vi) fixed maximal ideals of C (X) and C* (X), their characterizations, Structure spaces.
vii) Topological groups.

Also there is a scope, for applying the acquired knowledge of the above methods/ tools of Advanced Topology II, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. W. S. Massey – Algebraic Topology.
2. W. S. Massey – Singular Homology Theory.
3. E. H. Spanier – Algebraic Topology.
4. B. Gray – Homotopy Theory An Introduction to Algebraic Topology.
5. C. R. Bredon – Geometry and Topology.
6. Richard E. Chandler, Hausdorff Compactifications (Marcel Dekker, Inc. 1976).
7. L. Gillmen and M. Jerison, Rings of Continuous Functions (Von Nostrand, 1960).
8. Topological Structures (Halt Reinhurt and Winston, 1966).